# Optimization of frameworks by means of FEM use

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**ABSTRACT**: The paper presents an application of an optimization procedure for a mass optimization of a welded framework of a special tractor trailer designed for transport of seeding machines. The used optimization procedure, so-called Fully Stressed Design (FSD), is based on an indirect approach utilizing optimum criteria. The aim of the optimization was to achieve the lowest possible mass of the construction taking into consideration the allowed resistance. As the paper shows, on the basis of the optimization procedure we achieved more than 35% savings of the material mass. In this way the optimization procedures have an important and unreplaceable role in practice, because they result in significant material savings keeping the required resistance and consistency parameters of constructions. For the resistance, consistency and optimization analyses there was used the programmed system Pro/MECHANICA in an integrated mode with CAD product Pro/ENGINEER.

**Keywords**: framework; welded construction; Finite Element Method (FEM); optimization procedure; Fully Stressed Design (FSD)

Success of an intended product, including an agricultural machine, can be significantly influenced already in the phase of its designing. It has been proved that in this phase it is possible to influence up to 70% of the production costs. Designers (Medvecký et al. 1999) can directly influence complex costs (costs of production, operation, liquidation, recycling, etc.), the complex derived value and complex time (time of production, assembly, delivery, disassembly, liquidation). That is why the contemporary new design methods and philosophies play very important roles.

Current trends of designing increasingly tend to a broad use mainly of the CA technologies. In the design process of a product system approaches assisted by 3-D modelling of virtual prototypes are used. An integral component of the design proposal are also consistency analyses mostly done by means of the Finite Element Method (FEM) with subsequent optimization. A scheme of the conception of the Computer Aided Engineering (CAE) at a machine element designing is presented in Fig. 1.

In the process of dimensioning elements (subgroups, groups) it is necessary to ensure also the required construction reliability and safety. Requirements for the high machine reliability, demanding production process, production safety force the engineers to design every component as an optimal one utilizing optimizing procedures including the best design solution based on the set of entry conditions and required parameters of the relevant component. These conditions define the optimization conditions, e.g. a satisfactory running of the construction in some upper and lower limit of response, decrease of the mass of the component, definition of the parameters which are either constant or variable during the process of optimization, etc. In designing of complicated constructions it is necessary to optimize both, individual components as well as the whole construction.

In recent years (Žmindák et al. 2000) the use of special software products has significantly increased. A remarkable progress has accompanied also the optimization methods development (Baier et al. 1994). Traditional manual mathematical methods, which did not even respect physical merits of the problem, give way to significantly more effective methods among which the Finite Element Method (FEM) plays a very important

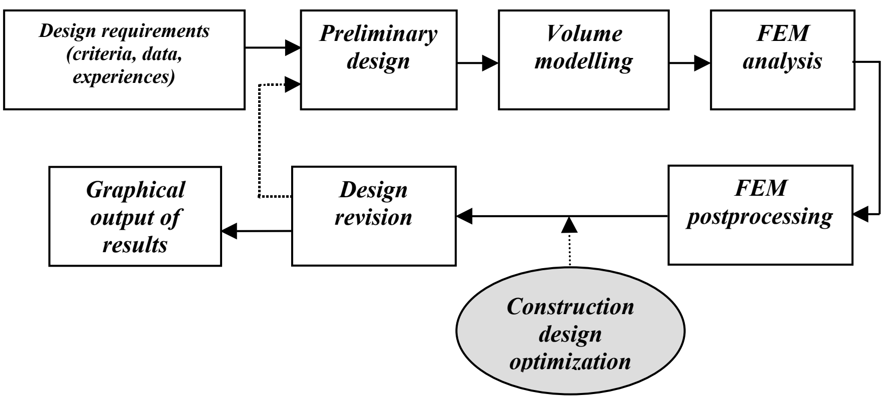


Fig. 1. Conception of CAE at a machine element designing

role. There are many application variants of this method (KOLÁŘ et al. 1997) and currently it is used by the majority of software companies involved in design. They also are a main part of the system CAD-CAE structure with excelently managed graphical facilities.

Optimization processes (ŽMINDÁK et al. 2000) can already be studied in the first works of Maxwell (1869) and later of Michell (1904). They present basic theories for the optimal placement of bars in a bar construction with a minimal mass. In the fifties of the last century Hempton (1958) and Chan (1960) further continued to develop Mechelle type of constructions. During the World War II the main attention was paid to the basic aircraft construction elements optimal design searching for their minimal mass. Two early papers were published by Smith (1943) and Zahorski (1944). In the second half of 40s and at the beginning of 50s the attention was focused on the development of methods for minimization of the mass of those aircraft elements for which buckling limitations were required. Shanley (1952) and Gerard (1956) published results of their work in this field. Most of these problems can be expressed by means of linear programming. First works presenting solutions of these problems were published by Heyman (1951), Foulkes (1954), Prager (1956) and Livesley (1956). Later Prager (1958) and Livesley (1959) dealt with the issues of the optimal design considering plastic collapse incidented by multiple loading and also some other types of loading. This type of optimization was used mainly in building industry. Since 1960 many optimization problems have been solved by means of mathematical programming. New optimal designing conceptions started to be developed due to Schmit (1960). These conceptions link up the Finite Element Method with mathematical programming. In most of them the goal function is the mass of the construction. At this time the construction optimization was divided into two main directions:

- system level optimization,
- element level optimization.

The first mathematical programming applications in optimization of armed shell constructions were published by Kircher (1968). These were followed by Morrow and Schmit (1968) who presented an optimal design of armed cylindrical shells. In the 60s several programmes using Schmit conception were created mostly in FORTRANE (Gallagher, Bell Aerosystems, 1964; Karnes and Focher, Boeing, 1968). Most of these programmes were based on the deformation formulation of FEM, and Zoutendijk algorithm of feasible directions was implemented to find the minimum.

The beginning of the 70s showed that Schmit's concept is not suitable for resolving of a lot of practical problems, because some calculations needed too much computer time. In order to find more effective optimization processes for large-scale systems with a great number of finite elements an aproximation of the goal function and its limitation through Taylor series were utilized. In this way the problem of the mathematical programming was transformed into linear programming problems sequence resolving. In connection with these new conceptions, a new division of optimal designing processes into three main categories have occured:

- intuitive procedures,
- procedures based on the optimum criteria,
- procedures using mathematical programming algorithms.

Further new more effective methods for minimization of the mass of constructions were developed at the beginning of the 80s. These methods combine approximation conceptions with dual formulations. The dual conception was then applied also to a discrete optimization which became an important part of the designing process of constructions from composite materials.

The objective of the paper is to show the possibilities of optimization process applied to one of the very frequent tasks – optimization of frameworks. There is used an indirect approach utilizing optimum criteria, so-called Fully Stressed Design (FSD). We point at the important and unreplaceable role which these processes have in practice, because they lead to significant material savings keeping the required resistance and consistency parameters of constructions. For the resistance, consistency and optimization analyses there was used the programmed system Pro/MECHANICA in an integrated mode with CAD product Pro/ENGINEER which has been used at our department already for several years.

## MATERIALS AND METHODS

Frameworks optimization belongs to the first optimization tasks. Its key point is to find a construction which requires the lowest costs and fulfils all necessary conditions. In practice this task is simplified into finding of a construction requiring the smallest amount of material (a construction of the minimal mass). In our case – the case of chassis, we use a section optimization of the framework. The solution of this optimization task itself assumes that the particular components are prismatic and every section represents one variable of the scheme. To reduce the number of limitations during the optimization process, we take into consideration only the critical limitations, i.e. limitations in the forms of equations. Consequently, a problem of the optimal scheme in general can be formulated on the basis of the non-linear programming: there has to be found a vector of the variables of the scheme:

$$\boldsymbol{X^T = [X_1, X_2, ..., X_n]} \qquad (1)$$

so that the goal function:

$$\boldsymbol{Z = F(X) \rightarrow \min} \qquad (2)$$

respecting the limitations:

$$\boldsymbol{g_j(X) \leq 0,\ j = 1, ..., n} \quad \text{– number of the limitations} \qquad (3)$$

The goal function $\boldsymbol{Z}$ can stand for the mass or price of the construction expressed through the scheme variables $\boldsymbol{X}$. In the case that the scheme variables are cross sections, it is possible to express the mass in the form:

$$Z = \sum_{i=1}^{n} l_i X_i = l^T X \qquad (4)$$

where $l$ is a vector of the variables of the scheme. Limitations are set for the cross-sectional area of the beams, for the joint displacements and stresses in section. Then:

$$X^D \leq X \leq X^H \qquad (5)$$

$$U^D \leq U \leq U^H \qquad (6)$$

$$\sigma^D \leq \sigma \leq \sigma^H \qquad (7)$$

Displacements $U$ and stresses σ are usually implied non-linear functions of the scheme variables $X$. For the given values $X$ the corresponding values of the displacements $U$ and stresses σ can be calculated through the deformation or force method. Then the optimization model itself can be created on the basis of non-linear or linear programming, geometric programming or on the basis of the criteria of the optimum.

In the following part we will present and use an indirect approach based on the optimum criteria which is suitable for resolving of statically determined frameworks. A design with the minimal mass can then be reached either by the use of simple recurrent formulae obtainable from Kuhn-Tucker optimum conditions, or by the use of various approximations. A recognized procedure (KOMPIŠ et al. 1991) is the so-called Fully Stressed Design (FSD). This procedure has the following practical advantages:

- Engineering experiences have proved that there is usually one good design, and it is the one in which all the beams are loaded in a fully stressed mode.
- It gives optimum results for the statically determined cases.
- It is more effective than the other methods.
- It can serve as a starting point for further more accurate methods of non-linear programming.

In the framework construction optimizing we applied just the FSD method which proved itself to be very suitable for solving practical tasks. If we assume that the suggested variable $X_i$ will be the section area, then the equivalent stress can be calculated according to the Mises formula (HMH):

$$\sigma_{ekv} = \sqrt{\sigma_x^2 + 3 \cdot [\tau_1^2 + (\tau_2 + \tau_3)^2]} \qquad \text{or}$$

$$\sigma_{ekv} = \sqrt{\sigma_x^2 + 3 \cdot [(\tau_1 + \tau_3)^2 + \tau_2^2]} \qquad (8)$$

Then the iterative formula can be written in the form:

$$X_i^k = \frac{\sigma_{iekv}^{k-1}}{\sigma_{idov}} \cdot X_i^{k-1} \qquad (9)$$

This formula expresses the crucial progress of the solution. In the case of the beam component we use more section characteristics what causes some complications in the choice of design variables.

## RESULTS OF OPTIMIZING

The load and clearance capacity of the seeding machine, for transport of which the tractor trailer was designed (its virtual model see Fig. 2), are the basic input values for resistance analysis. In order to calculate the load we used the mass of the seeding machine ($m$ = 2,000 kg) and the operation conditions.

The resulting load acting on the frame:

$$F_V = K_D \cdot m \cdot g = 2.25 \,.\, 2{,}000 \,.\, 10 = 45{,}000 \qquad (10)$$

where: $K_D$ ($K_D$ = 2.25) – the impact factor (which defines the dynamic conditions),
$g$ ($g \approx 10$ m/s$^2$) – the gravity acceleration.

The load is evenly distributed onto the whole construction, see the calculation model in Fig. 3. The framework is created on the basis of the conceptual design from the rolled sections as a welded construction. The suggested material used for the rolled sections is the steel 11523.1 having the following mechanical properties (FIALA et al. 1990):

- Modulus of elasticity $E = 2.1 \,.\, 10^5$ MPa,
- Yield point $R_e$ = 343 MPa,
- Breaking strength $R_m$ = 510 ÷ 628 MPa,
- Specific mass $\rho = 7.85 \,.\, 10^{-3}$ kg/mm$^3$,
- Poisson's material ratio $\mu = 0.3$.

To create the volume model and conseqently to do its analysis the software systems Pro/ENGINEER and Pro/MECHANICA were used. In the resistance and consistence analysis the program package works on the basis of the **Method of Geometrical Elements** which is one of the progressive methods of the continuum mechanics. Both products were developed by the Parametric Technology Corporation in the U.S.A. The framework was divided into the volume and shell elements (reinforcements). Constructions were defined into three points. In the place of the rod there are reduced three adjustable degrees of freedom in the direction of coordinate axes. On the axle in the point of the fixation of the axle there are two binding points. In both of them four degrees of

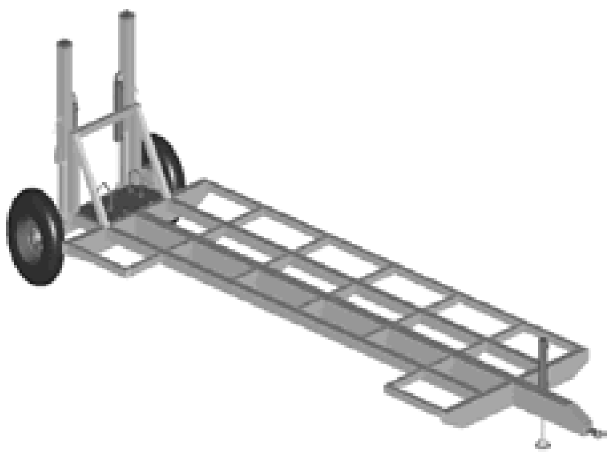

Fig. 2. Virtual model of the designed tractor trailer aimed for transport of seeding machines

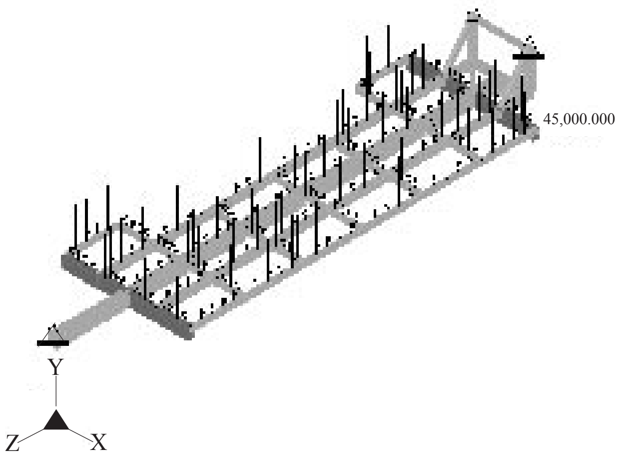


Fig. 3. Calculated model of a framework

freedom are reduced (shifts in the directions of the axes $y$ and $z$, and possible rotations around the axes $x$ and $z$). Totally 11 degrees of freedom were reduced from the volume model (see Fig. 2).

The model, defined in this way, was exposed to a static analysis and subsequent optimization for minimal mass of construction. The static analysis of the initial construction design showed that the stresses ($\sigma_{max}$ = 71.25 MPa) determined on the construction predetermine application of the optimization aimed at the reduction of the total mass of the construction keeping its dynamic safety and sufficient consistency. Initial mass of the intuitively designed frame of the chassis was 1,368.9 kg.

For each beam of the frame 10 sections of the closed thin-walled profile were defined, and the optimization process itself was done in 10 iterations. As an optimal solution of the framework construction there were selected sections in which the achieved stress was $\sigma_{idov}$ = 150 MPa (Fully Stressed Design – FSD).

The survey of the results of optimization calculations is depicted in graphs in Fig. 4, where also the dependence of the construction mass change and the reduced stress change from the iteration step can be seen. This process made it possible to reduce the total mass of the welded framework construction to 878.12 kg, i.e. the achieved mass cut-down was 490.78 kg what represents the material mass savings of 35.85%.

## CONCLUSIONS

Comparison of the masses before and after the optimization makes it possible to state that after the application of the so-called Fully Stressed Design (FSD) optimization process the best solution seems to be the use of predefined sections in the 5th iteration step for which the section of the main beam **UE 20 STN 42 5571** refers. Analogically, it would be possible to determine also the sections of other beams.

After the comparison of the masses of the optimized alternative with the initial solution it is possible to express the conclusion that in frame of the chassis the mass savings are $m_U$ =1,368.9 – 878.12 = 499.78 kg, what is more than **35%.** Taking into account the fact that more pieces of it will be made, then, under the continually increasing prices of materials, this is not a negligible value.

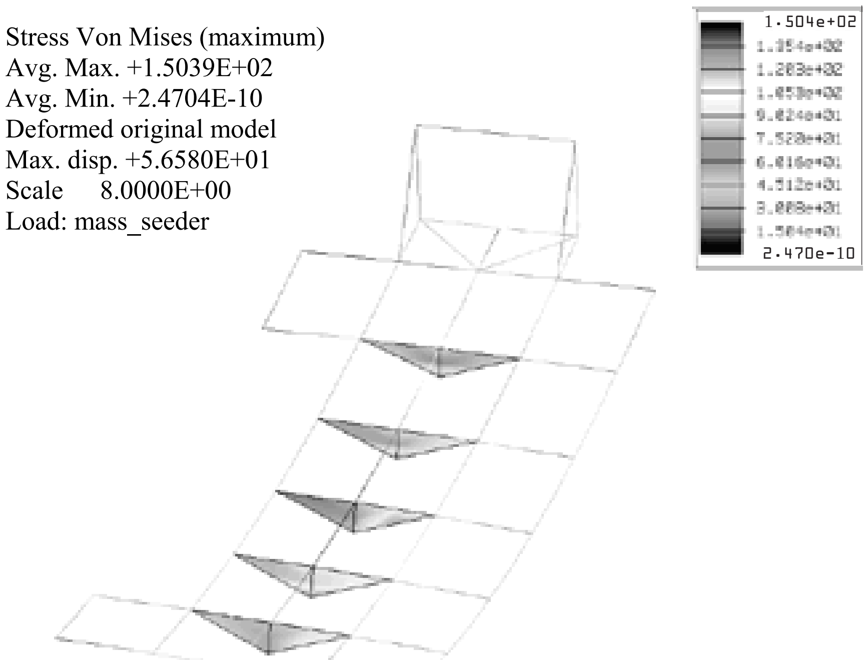


Fig. 4. Behaviour of reduced stress value (Von Mises) – optimized framework

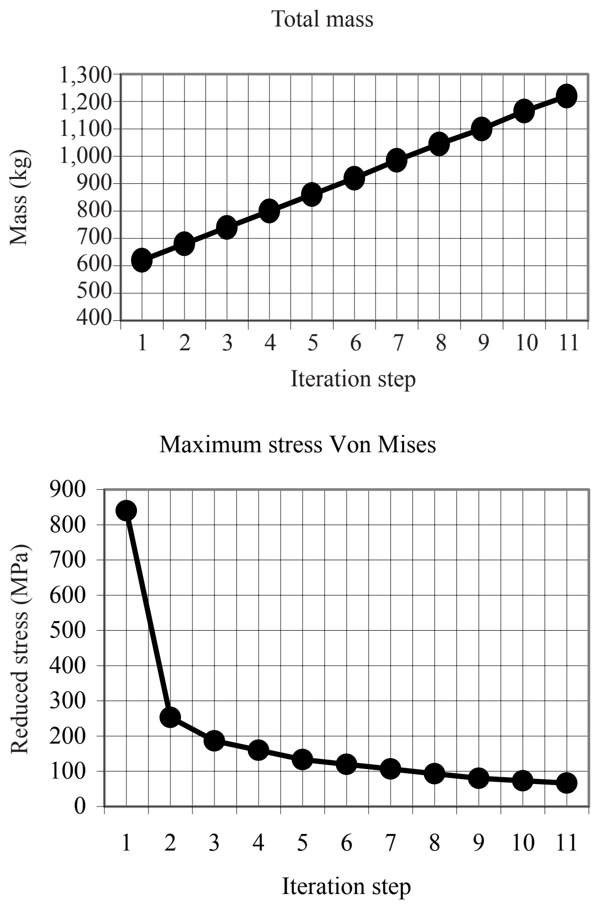


Fig. 5. Results of optimization calculations

Thus the optimization process proved that by using the program system **Pro/MECHANICA** it is possible to change the cross profiles of the framework beams in such a way, that their minimal mass (the lowest price) is achieved at a sufficient resistance and construction consistency.

**References**

BAIER H., SEEßELBERG CH., SPECHT B., 1994. Optimierung in der Strukturmechanik. Wiesbaden, Vieweg: 267.

FIALA J., BEBR A., MATOŠKA Z., 1990. Strojnické tabulky 1. Praha, SNTL: 877.

KOLÁŘ V., NĚMEC I., KANICKÝ V., 1997. FEM – Principy a praxe metody konečných prvků. Praha, Computer Press: 401.

KOMPIŠ V. et al., 1991. Optimalizácia konštrukčných návrhov v spojení s MKP. Bratislava, Edičné stredisko Slovenskej technickej univerzity: 304.

MEDVECKÝ Š. et al., 1999. Základy konštruovania. Žilina, EDIS – vydavateľstvo Žilinskej univerzity: 599.

ŽMINDÁK M., SÁGA M., TVARŮŽEK J., HUSÁR Š., 2000. Optimalizácia mechanických sústav. Žilina, EDIS – vydavateľstvo Žilinskej univerzity: 261.

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## Optimalizácia rámových konštrukcií použitím MKP

**ABSTRAKT**: V príspevku prezentujeme aplikáciu optimalizačného postupu pri hmotnostnej optimalizácii rámovej zváranej konštrukcie špeciálneho podvozku na prevoz sejačky. Použitá metóda, tzv. návrh na plný napäťový stav (Fully Stressed Design – FSD), je založená na nepriamom prístupe využívajúcom kritériá optimálnosti. Cieľom optimalizácie bolo teda dosiahnuť najmenšiu hmotnosť konštrukcie so zohľadnením dovolenej pevnosti. Ako je uvedené v článku, nami dosiahnutá úspora materiálu na základe realizovanej optimalizácie predstavuje viac ako 35 %. Optimalizačné postupy teda zaujímajú dôležité a nezastupiteľné miesto v praxi, pretože vedú k veľkým úsporám materiálu pri zachovaní požadovaných pevnostných a tuhostných parametrov konštrukcií. K realizácii pevnostných, tuhostných a optimalizačných analýz bol aplikovaný programový systém Pro/MECHANICA v integrovanom móde s CAD produktom Pro/ENGINEER.

**Kľúčové slová**: rámová konštrukcia; zváraná konštrukcia; metóda konečných prvkov (MKP); optimalizačné metódy; návrh na plný napäťový stav

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