Determination of elastic parameters of grain with oedometric and acoustic methods

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ABSTRACT: Values of modulus of elasticity *E* and Poisson's ratio ν were determined with two methods adopted from geotechnique. First approach used was a method proposed by SAWICKI (1994). This type of examination was applied to estimate values of *E* and *ν* for wheat and rapeseed beddings for five levels of moisture content ranging from 6% to 20%. Modulus of elasticity *E* of wheat was found to decrease with an increase in moisture content. With the second method values of *E* were determined based on measurement of shear wave velocity. Tests were performed for bedding of wheat and rapeseed under equilibrium moisture content. Values of modulus of elasticity were found to depend of hydrostatic pressure and were higher then those determined in uniaxial compression test.

Keywords: grain; wheat; rapeseed; elastic parameters; uniaxial compression; shear wave velocity

Numerous food and agricultural materials are stored and processed in a particulate form. Cereal grains and rapeseed are also important raw materials for food industry. Properties of the material serve as parameters for engineers designing storage systems or processing plants.

Mechanical properties of grain bedding depend on properties of individual grains, friction between particles, interparticle contact geometry and load history. Recently demand of industrial practice resulted in revision of several silo design codes, which included standardization of methods for determination of mechanical properties of granular materials. European Standard Eurocode 1 (1996) and Polish Standard (DYDUCH et al. 2000) recommend to determine properties under load condition which are similar to the operation loads. Some properties are determined using shear test (Jenike tester or triaxial compression test) other using uniaxial compression test.

The aim of performed experiments was to evaluate and comprise values of mechanical constants characterizing elastic deformation (modulus of elasticity *E* and Poisson's ratio) of rapeseed and wheat bed determined with two methods adopted from geotechnique. First approach used was a method proposed by SAWICKI (1994) who used first phase of unloading in uniaxial compression experiment to determine elastic constants. This type of examination was applied to estimate values of *E* and ν for wheat and rapeseed beddings for five levels of moisture content ranging from 6% to 20%. With the second method values of *E* were determined based on measurement of shear wave velocity (TIMOSHENKO, GOODIER 1970). Tests were performed for bedding of wheat and rapeseed under equilibrium moisture content.

MATERIALS AND METHODS

Values of basic mechanical constants characterizing elastic deformation modulus of elasticity *E* and Poisson's ratio ν were determined with two methods adopted from geotechnique. First approach used was a method proposed by SAWICKI (1994) who distinguished

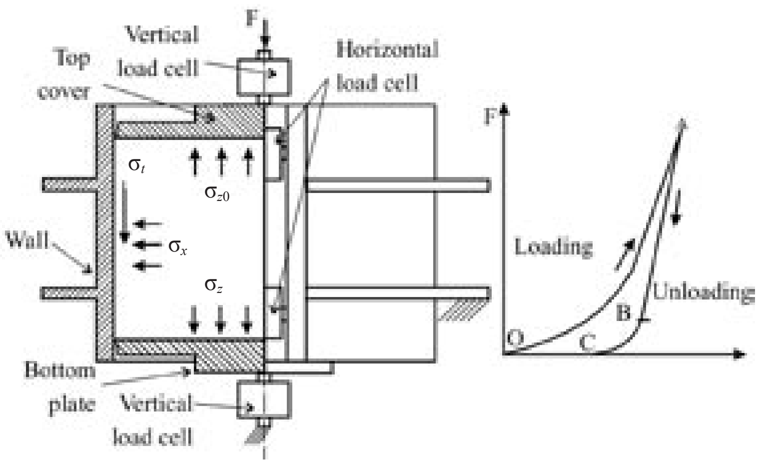

Fig. 1. Uniaxial compression test

two phases of the unloading in uniaxial compression experiment. The first phase is characterized by a purely elastic deformation and can be used for determination of elastic constants. This type of examination was applied to estimate values of E and ν for wheat (variety Begra) and rapeseed (variety Mar) beddings for five levels of moisture content ranging from 6% to 20%. Uniaxial compression test was used as shown in Fig. 1 (HORABIK, MOLENDA 2000).

The wall of the uniaxial compression tester consisted of two semicircular halves cut along the axis. The two semicircular halves were connected with four load cells installed in pairs on the two connection lines, restoring the cylindrical shape of the wall. One half of the wall rested directly upon the base. Bottom and top plates of the chamber transmitted the vertical load through the load cells. The experimental set allowed for determination of the mean lateral pressure σ_x, the mean vertical pressure on the bottom σ_z, and the mean vertical pressure on the top plate σ_{z0}. Surface of the wall was smooth while the surfaces of the top and bottom plates were rough. The granular material was poured into the test chamber, without vibration or other compacting action. The sample was 80 mm high and 21 cm in diameter. The bedding was loaded to the reference vertical stress σ_{z0} of 100 kPa using universal testing machine. The top cover of the apparatus was moving down with a constant speed of 0.35 mm/min, while the displacement was measured with inductive sensor having accuracy of 0.01 mm. Next unloading took place with the same speed of deformation until to the 0 kPa of stress level was reached. Experiments were performed in three replications.

Two phases of the unloading can be observed (Fig. 1). The first phase is characterized by a purely elastic deformation and was used for determination of elastic constants, the modulus of elasticity E and Poisson's ratio ν. The second stage of unloading is characterized by both elastic and plastic deformations. It is assumed that the material reversible response is governed by Hook's law:

$$\varepsilon^e_x = \frac{1}{E}\left[(1 - \nu)\,\sigma_x - \nu\sigma_{z0}\right] \quad (1)$$

$$\varepsilon^e_z = \frac{1}{E}\left[\sigma_{z0} - 2\nu\sigma_x\right] \quad (2)$$

During the first phase of unloading (path AB) the sample shows linear reaction which is characteristic for elastic deformation. Assuming that $\varepsilon_x = \varepsilon^e_x = \varepsilon^p_x = 0$ from (Eq. 1) $\frac{\sigma_x}{\sigma_z} = \frac{\nu}{1-\nu}$

is obtained and applying assumption that $\varepsilon_z = \varepsilon^e_z$ to (Eq. 2) may be expressed as below:

$$\varepsilon_z = \frac{\sigma_{z0}}{E}\left(1 - \frac{2\nu^2}{1-\nu}\right) \quad (3)$$

Elastic constants were determined using experimental results from linear phase of unloading. The ratio of horizontal stress σ_x to vertical stress σ_{z0} was assumed to be constant (elastic state of stress) and a slope A of a straight line, $A = \sigma_x/\sigma_{z0} = \nu/(1 - \nu)$, was estimated using linear regression procedure applied to experimental values of stresses. Then, with A estimated, values of Poisson's ratio ν were calculated as:

$$\nu = \frac{A}{1 + A} \quad (4)$$

Values of modulus of elasticity E were estimated using relationship $\varepsilon_z(\sigma_{z0})$ (Eq. 3) with experimental values of ε_z and σ_{z0}, and ν determined as described above.

Linear elastic theory allows to calculate elastic parameters on the basis of measured both shear and longitudinal wave velocities (LIPINSKI 2000). Parameters obtained on the basis of equation *o* theory of elasticity refer to small strain of the order of 10^{-5} and therefore shear modulus G which is most required parameter in engineering application, is often termed initial modulus G_o or stiffness modulus G_{max}. Initial modulus can be calculated if only shear wave velocity V_s and mass density ρ are known as: $G_{max} = \rho V_s^2$ (TIMOSHENKO et al. 1970). And then modulus of elasticity of the material can be calculated (JASTRZĘBSKI et al. 1985). This method was applied to evaluate state of cohesionless soils by Lipiński but was never before used in determination of properties of agricultural granular materials. LIPIŃSKI (2000) determined shear wave velocity for Ottawa sand in a range of hydrostatic pressure from 50 to 400 kPa. He stated that velocity of sound increase from 150 to 300 m/s. With this method values of E were determined based on measurement of shear wave velocity for bedding of wheat and rapeseed under equilibrium moisture content. Experiments were performed under different hydrostatic pressure in triaxial chamber fitted with piezoelements in top and bottom covers of the apparatus that allowed for generating and recording of acoustic shear waves (Fig. 2).

The properties were determined under load condition similar to those encountered for bins in practical operation regime. The material was poured into the test chamber, without vibration or other compacting action. Maximum normal pressure applied in uniaxial compression test (of 100 kPa) corresponded to pressure produced by approximately 14 m high column of grain. Measurements of shear wave velocity in grain bedding were conducted under hydrostatic pressure ranging from 10 to 90 kPa that was increased in 10 kPa steps in each experiment. The sample was 80 mm high and 21 cm in diameter in uniaxial compression test and 150 mm high and 70 mm in diameter in hydrostatic chamber.

RESULTS

Uniaxial compression test

Fig. 3 shows relationships of vertical stress σ_{z0} and total vertical strain ε_z for loading – unloading cycles for rapeseed of four levels of moisture content. First part of

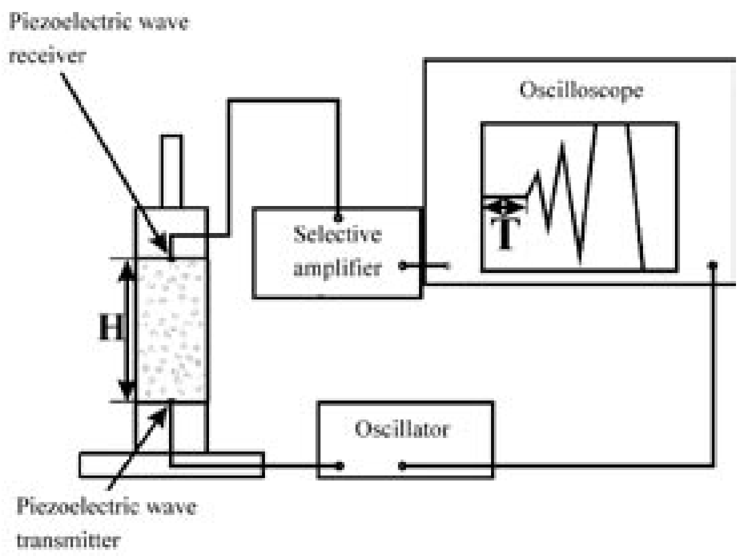

Fig. 2. Hydrostatic chamber constructed to determine shear wave velocity

the loading curve reflects consolidation of the sample with translation and rotation movements of particles, but without deforming them. Second, steeper part of the curve shows an increase in the plastic and elastic stresses in the sample associated with deformation of particles.

During this phase of loading deformation takes place mainly in contact areas between grains. Variation of moisture content resulted in large differences between experimental stress – strain relationships, which were reflected in values of model parameters.

Results of estimation of elastic constants are presented in Table 1. Modulus of elasticity of wheat ranged from 11 MPa to 22.4 MPa, while in the case of rapeseed it ranged from 6.6 MPa for 15% moisture content to 9 MPa for 6% moisture content.

For such a height of bedding Polish Standard – PN-89/B-03262 (1989) and proposition of its amendments (DYDUCH et al. 2000) recommend applying modulus of elasticity for grain of 20 MPa, thus being in good agreement with our results. SAWICKI and ŚWIDZIŃSKI (1998) obtained E of wheat of 96 MPa. The reason for this difference is probably distinctly higher maximum pressure of 800 kPa applied in the tests of those authors. Sample of SAWICKI and ŚWIDZIŃSKI (1998) was 40.5 mm high and 75.5 mm in diameter, thus being distinctly smaller than that of ours, which may also contribute to observed discrepancy in tests results. Values of modulus of elasticity for wheat reported by various authors ranged widely – from 0.7 MPa to 70 MPa (THOMPSON, ROSS 1984; ZHANG et al. 1988). Such a large range of variation is probably a result of differences in experimental methods, levels of applied pressures and experimental material itself.

Maximum value of Poisson's ratio ν of 0.22 was obtained in the case of wheat of 10% of moisture content while minimum of 0.18 was found for wheat of 12.5%. Values of ν in the case of rapeseed were found in a range from 0.10 to 0.24 and increasing with a decrease in moisture content from 15% to 6%. SAWICKI and ŚWIDZIŃSKI (1998) found ν of 0.27 in the case of dry wheat. ZHANG et al. (1988) reported Poisson's ratio of 0.29 for dry wheat. Polish Standard – PN-89/B-03262 (1989) recommends that value of Poisson's ratio for

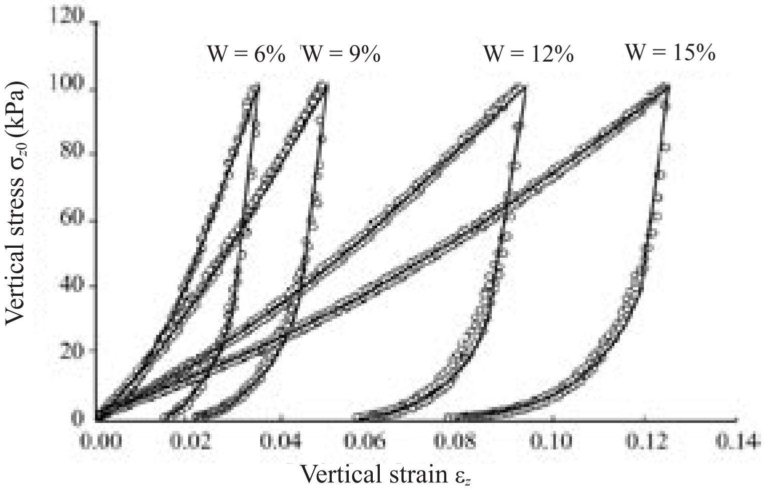

Fig. 3. Experimental data and fitted $\sigma_{z0}(\varepsilon_z)$ relationships for rapeseed of four levels of moisture content

Table 1. Parameters of rapeseed and wheat determined in experiments

Grain	Moisture content (%)	Modulus of elasticity E (MPa)	Poisson's ratio ν
Wheat	10.0	22.4 ± 4.6	0.22 ± 0.01
	12.5	22.2 ± 4.4	0.18 ± 0.02
	15.0	19.3 ± 2.5	0.20 ± 0.03
	17.5	17.2 ± 3.6	0.20 ± 0.01
	20.0	11.1 ± 1.1	0.19 ± 0.01
Rapeseed	6.0	9.0 ± 0.6	0.24 ± 0.03
	9.0	8.7 ± 0.8	0.17 ± 0.02
	12.0	7.1 ± 0.6	0.16 ± 0.01
	16.0	6.6 ± 0.9	0.10 ± 0.01

$E = 2\rho V_s^2(1 + \nu)$		
Hydrostatic pressure (kPa)	*E* (MPa) evaluated for ν = 0.2	*E* (MPa) evaluated for ν = 0.4
Rapeseed		
10	22.6 ± 1.9	26.4 ± 2.2
20	28.9 ± 0.9	33.8 ± 1.1
30	34.6 ± 0.6	40.4 ± 0.7
40	38.7 ± 0.5	45.1 ± 0.5
50	42.5 ± 1.3	49.6 ± 1.6
60	45.4 ± 1.6	53.0 ± 1.8
70	49.4 ± 1.8	57.6 ± 2.1
80	53.5 ± 0.7	62.4 ± 0.9
90	57.5 ± 1.0	67.1 ± 1.2
Wheat		
10	20.2 ± 3.3	23.6 ± 3.8
20	26.8 ± 4.6	31.3 ± 5.4
30	35.8 ± 6.5	41.8 ± 7.5
40	41.8 ± 9.4	48.8 ± 11.0
50	52.9 ± 8.5	61.8 ± 9.9
60	60.6 ± 7.2	70.7 ± 8.4
70	65.6 ± 4.2	76.5 ± 5.0
80	71.1 ± 7.6	82.9 ± 8.9
90	75.4 ± 7.4	88.0 ± 8.6

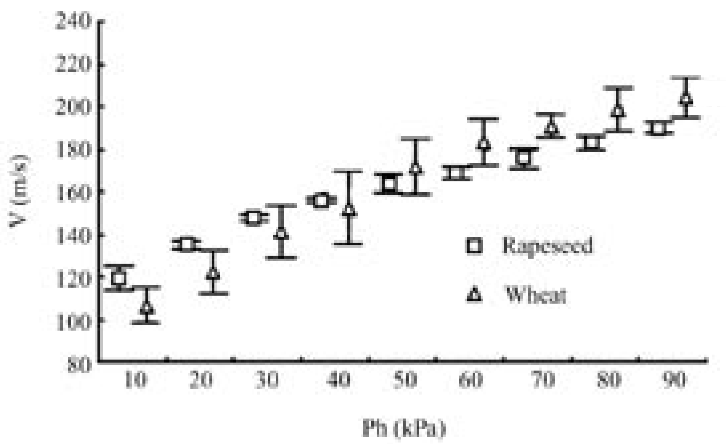

Fig. 4. Relationship between hydrostatic pressure and shear wave velocity determined for rapeseed and wheat

grain of 0.4 should be used for design. The standard does not contain any information about the influence of moisture content on the two elastic constants of the material.

Acoustic examination

Velocity for wheat was found in a range from 106 m/s at 10 kPa of hydrostatic pressure to 204 m/s at 90 kPa. In the case of rapeseed velocity ranged from 119 m/s to 190 m/s (Fig. 4). Values of modulus of elasticity for wheat (from 23.6 to 88.1 MPa) were found varying in a wider range then those for rapeseed (from 26.3 to 67.0 MPa) for hydrostatic pressure in a range from 10 to 90 kPa (Table 1). This is probably a result of difference in a ranges of sample deformation which were very small (strain of 10^{-5}) in shear wave velocity estimation while in uniaxial compression tests the level of 12% of strain was applied.

CONCLUSIONS

Values of material constants determined in uniaxial compression test were found dependent on species and on moisture contents of the material.

Values of E decreased with an increase in moisture content.

Values of modulus of elasticity, determined in uniaxial compression test in a range from 6.6 MPa to 32.6 MPa, were close to those recommended by standards.

Values of Poisson's ratio (0.1÷0.24) were found lower than the value of 0.4 recommended by Polish Standard.

Shear wave velocity measured in hydrostatic chamber with acoustic method increased with an increase in hydrostatic pressure.

Values of modulus of elasticity E estimated based on measurement of shear wave velocity were found higher then those estimated with uniaxial compression test.

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Zjišťování parametrů elasticity zrna endometrickými a akustickými metodami

ABSTRAKT: Byly zjišťovány hodnoty modulů elasticity E a Poissonova čísla ν pomocí dvou metod převzatých z geotechniky. Prvním způsobem byla cesta navržená SAWICKIM (1994). Tento druh zkoušek byl aplikován za účelem zjišťování hodnot E a ν pro vrstvy pšenice a řepky pro pět hladin vlhkosti v rozmezí od 6 do 20 %. Bylo zjištěno, že se modul elasticity pšenice zmenšuje se zvyšujícím se obsahem vlhkosti. U druhé metody byly hodnoty E stanoveny na základě měření rychlosti smykových vln. Testy byly prováděny pro vrstvu pšenice a řepky o stejné vlhkosti. Bylo zjištěno, že hodnoty modulu elasticity závisejí na hydrostatickém tlaku a byly vyšší než hodnoty stanovené v jednoosých kompresních testech.

Klíčová slova: zrno; pšenice; řepka; parametry pružnosti; jednoosá komprese; rychlost smykových vln

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