Loading regimes and designing helical coiled springs for safe fatigue life

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ABSTRACT: Springs are loaded by harmonic forces very often. High cycles fatigue damage and failure can be found during its service loading. This paper shortly describes stress concentration factor for helical springs on the inner diameter of the spring wire and its evaluation by using FE Method. These results have been compared with correction functions published in the literature. The fatigue safety factor has been derived for three typical loading regimes of springs. All cases are demonstrated in the Haigh diagram. It has been showed that loading cases with constant operational prestress give lowest safety factor then the proportional or constant middle stress regimes.

Keywords: springs; fatigue of materials; loading regimes; safe fatigue life

Springs often work in the regime of cyclic loading forces. Because they can be exposed to tens, hundreds or millions of loading cycles during their service life, they are prone to material fatigue that can cause rupture. Springs exposed to this kind of load are most frequently designed for a constant fatigue limit. However, we do not find a typical fatigue limit for a number of materials of which springs are made; for this reason, springs are usually designed for a high-cycle fatigue limit. This paper deals with the methods for determining the safety of helical coiled springs working in various regimes of harmonic load in a high-cycle region.

DETERMINING THE LOAD OF HELICAL COILED SPRINGS

The most frequent loading and determination of stress conditions in helical coiled springs (pitch diameter D and pitch of spring s) are described in every basic literature dealing with the design of basic machine components. Let us summarize here for completeness that during the axial load of a spring by force F, the spring wire is exposed to stress in the considered cross section (perpendicular to the wire longitudinal axis) by the vector of moments, $\boldsymbol{M}$, and the vector of forces, $\boldsymbol{F}$. Vector $\boldsymbol{M}$, which can be resolved into two components, causes bending stress (component $\boldsymbol{M}_o$) and torsion stress (component $\boldsymbol{M}_k$). Analogously, the vector of force $\boldsymbol{F}$ causes stress in the considered cross section by shearing component $\boldsymbol{T}$ (or, as the case may be, by a tensile force $\boldsymbol{N}$), as indicated in Fig. 1.

Under the assumption that the lead angle α of the spring is small (in practice, up to 5 degrees), components $\boldsymbol{M}_o$, $\boldsymbol{T}$ and $\boldsymbol{N}$ can be neglected and the dominant stress of the spring wire arises from the torsion. The shear stress (wire of a circular cross section with diameter d) increases linearly with radius ρ and assumes the rated design value on the wire surface, which is described by the following well-known relations (see also Fig. 1):

$$\tau(\rho) = \frac{M_K}{J_p} \times \rho = \frac{F\,\frac{D}{2}}{\frac{\pi\, d^4}{32}} \times \rho$$

$$\tau(R) \equiv \tau_{\text{nom}} = \frac{M_K}{W_k} = \frac{F \times \frac{D}{2}}{\frac{\pi\, d^3}{16}} = \frac{8F \times D}{\pi\, d^3} \tag{1}$$

However, the surface shear stress is not the same everywhere as the consequence of the spring helical shape. It is well known that the limit shear stress is reached in the points lying at the inside diameter of the spring, as illustrated in Fig. 1.

This fact also affects (although to a small extent) the deformation of the spring. In the cited literature, but also in other manuals and standards, relations

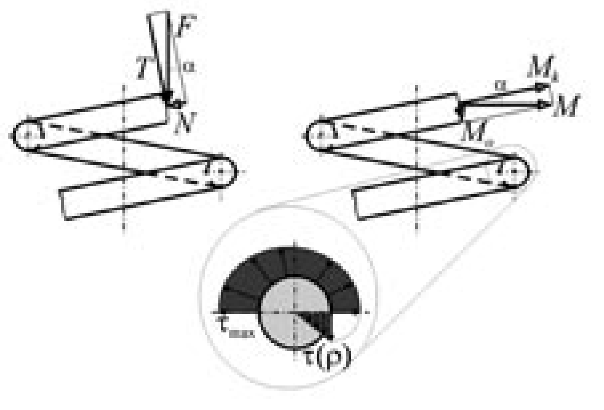


Fig. 1. Components of forces and moments loading the wire of a helical coiled spring

can be found that describe this shape concentration of the stress and the correction for deformation in quantitative terms.

It holds according to Young (1989)

$$\tau_{max} = \tau_{nom} \times \psi, \quad y_{actual} = y \times \phi \tag{2}$$

where: ψ , ϕ – correction coefficients for the stress and deformation.

These coefficients can be determined from approximation relations:

$$\psi = 1 + \frac{5}{4}\frac{d}{D} + \frac{7}{8}\left(\frac{d}{D}\right)^2, \qquad \phi = 1 - \frac{3}{16}\left(\frac{d}{D}\right)^2$$

Plots of the dependence of both correction coefficients are in Fig. 2.

We have verified the values of the dependence of the correction coefficient of the tension by numerical computations with the help of FEM. We used ABAQUS software for these calculations. The spring was modelled parametrically with the option of changing the D/d ratio for the chosen pitch of the spring. The model was formed by 8 nodal points C3D8R in the region of the entire coiling and 4 nodal elements C3D4 in the region of truncation (see Fig. 3).

The results of the dependence of the reduced tensions according to the HMH hypothesis for both limit alternatives of the modelled ratios (D/d = 10 and 2.1) are illustrated in Fig. 4. From the evaluation of the dependence of shear tensions in the transversal cross section (see Fig. 3), values of the correction coefficients were evaluated for tensions; they are compared in Fig. 2 with the dependence obtained according to the analytical formula. The difference between the calculated and theoretical values can be explained by the fact that the FEM calculations include the effect of the shear force and bending moment, which was neglected in analytical formulas.

FATIGUE LIMIT OF SPRINGS

When designing springs for a constant strength, it is necessary to know the fatigue limit, τ_C^*, for the considered spring. The fatigue limit can be found, for example, by examining the Wöhler fatigue curve (*S-N curve*). If the results of fatigue tests of real springs are not available directly, it is possible to estimate this value from fatigue limits σ_C or, as the case may be, τ_C of the Wöhler curves determined on etalon samples of a given material during the tensile stress (bending stress) or torsion stress. The basic type of test is carried out during symmetrically alternating load (with the coefficient of cycle asymmetry R = –1) on polished samples of small

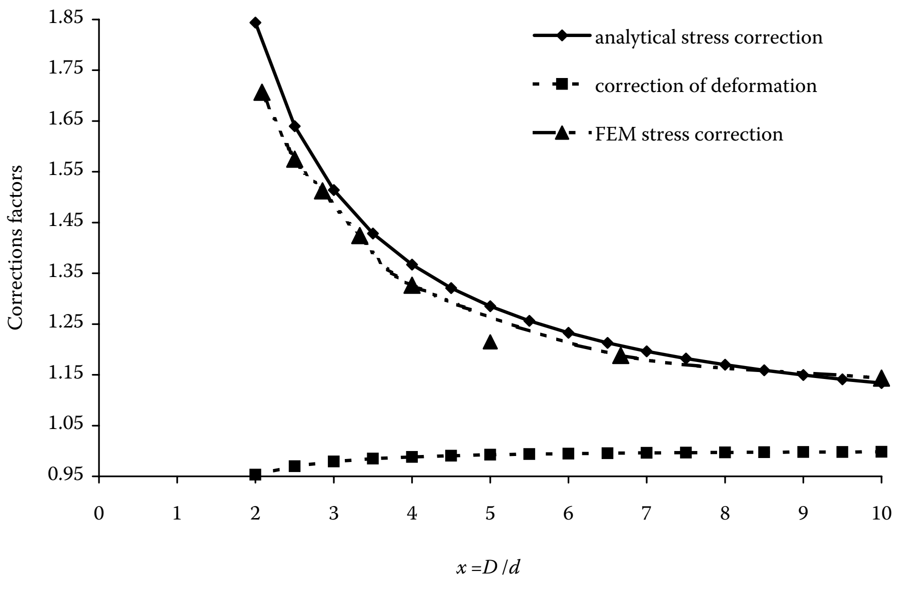


Fig. 2. Plots of the correction coefficients for the determination of the limit tension and actual deformation of a helical coiled spring depending upon its dimensions

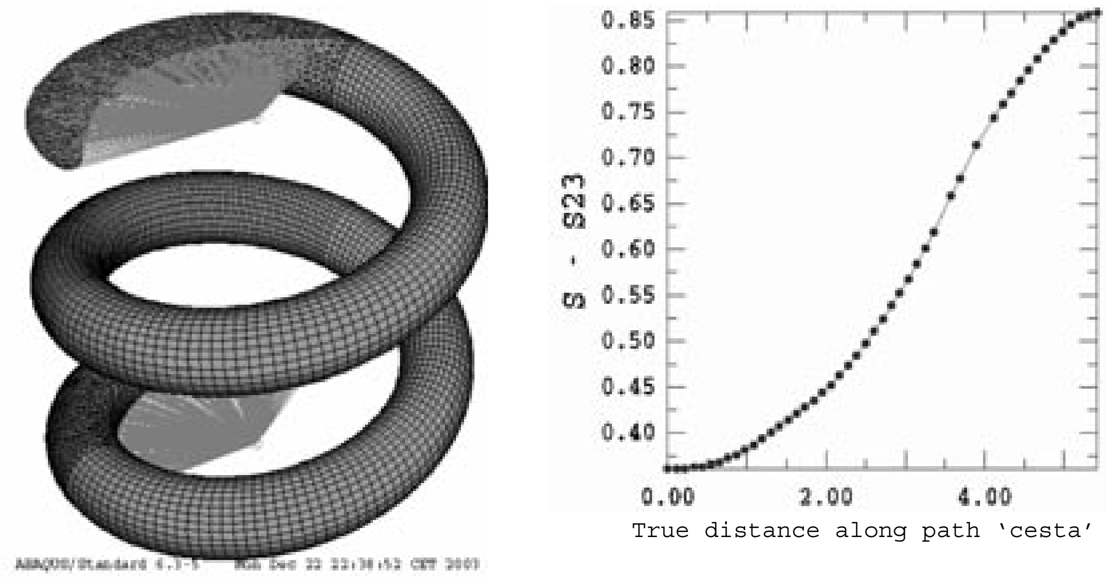


Fig. 3. Parameterized grid for FEM calculations and data collection in the transversal cross section along the wire circumference

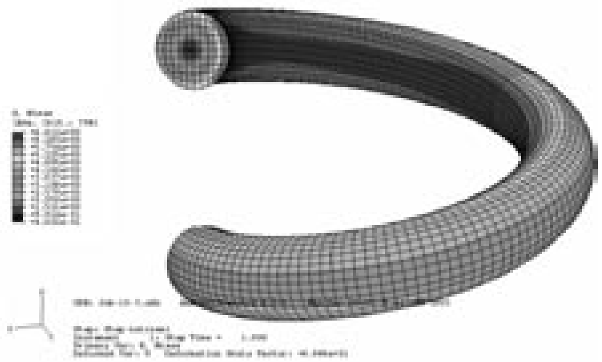
diameters. Because many materials used for springs (special steel, plastic, etc.) do not have a typical asymptotic value of the fatigue limit, it is necessary to count on a value for a high-cycle time-dependent fatigue limit (for example, for the number of cycles $N_C = 2\times10^6...10^7$ corresponding to the number of cycles of the working regimes). If the fatigue limits for the torsion stress are not available, they can be estimated from the values obtained for the tensile stress utilizing the validity of the "static" strength hypothesis, i.e. according to the relation

$$\tau_C = \frac{\sigma_C}{\lambda} \qquad (3)$$

where: $\lambda = 2$ for the hypothesis of maximum shear stress or $\lambda = \sqrt{3}$ for the HMH hypothesis (Huber-Mises-Hencky).

The above-indicated differences between the fatigue limits of a real spring and the etalon sample should be corrected by the size effect coefficient κ_v, surface quality coefficient κ_p or, as the case may be, coefficient κ_t of the material processing technology. Using this approach, we obtain $\tau_C^* = \tau_C \times \kappa_v \times \kappa_p \times \kappa_t$. Magnitudes of these coefficients can be found in fatigue manuals (for example, in Růžička et al. 1992). It should be emphasized that the surface treatment of the spring wire or, as the case may be, damage to its surface that may arise during spring winding has a significant effect on the resulting fatigue limit of springs; therefore, the surface quality should be checked carefully.

The effect of the mean component of tension τ_m is another important factor in the fatigue of material. Its effect can be taken into account by the limit points τ_A, τ_M on the curves in the Haigh diagram (see Fig. 5).



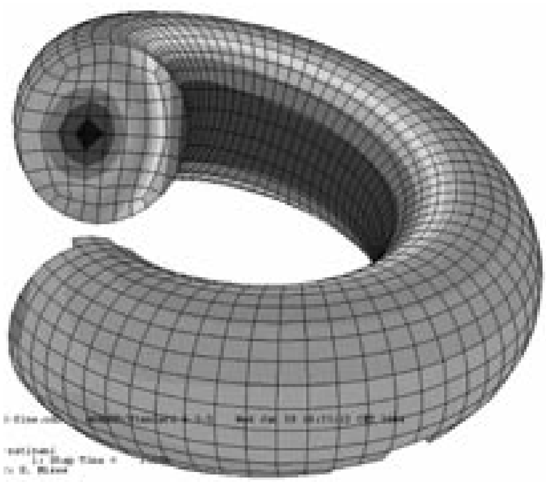


Fig. 4. Distribution of reduced tensions in the spring with parameter D/d = 10 and D/d = 2.1

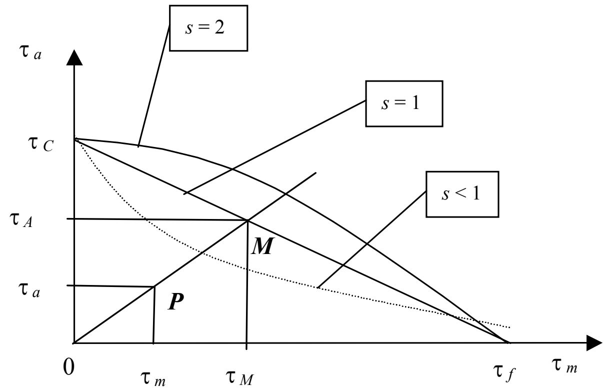


Fig. 5. Haigh diagram

The limit curves can be parameterized, for example, by the relation

$$\left(\frac{\tau_A}{\tau_C}\right)^s + \frac{\tau_M}{\tau_f} = 1 \tag{4}$$

when $s = 1$ is obtained for linear dependence, $s = 2$ for a parabola, etc. Values τ_f can be considered as the actual static breaking resistance in torsion; or the breaking resistance can be estimated from the value obtained during a tensile test analogously to Eq. (3).

WORKING REGIMES OF SPRINGS

The most frequent load of springs is either the basic static load (for example, by the machine mass or a preload generated by setting the working point of the spring mechanically), or a dynamic load causing fatigue load. The dynamic load is most frequently harmonic with frequency f given by the oscillation of the harmonic oscillator, y_G

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{y_G}}$$

where: y_G denotes the static compression (elongation) of the spring by the own weight of the oscillating mass m and g is the gravitational constant.

In the following sections, we will discuss just this type of loading. Stochastic loading is a more general case of spring loading caused by an exciting force of random values. However, the analysis of the service life for this region is outside the scope of this contribution. If the random peaks of tension amplitudes do not exceed the corresponding (equivalent) values of the amplitudes of the harmonic load for springs designed for unlimited service life, the safety of this spring will always be higher than during its own harmonic load.

The safety of springs is determined for the following three loading regimes: Individual loading regimes can be represented by a set of working points **P** that fill in the working curves (here, straight lines). The intersection of the loading curve with the limit fatigue curve in the Haigh diagram (see Fig. 5) determines the limit point of fracture **M**. If the loading force is parameterized by coefficient λ, it holds for the parameter in working point **P** that $\lambda = 1$. It holds that $\lambda = k$ on the limit curve when safety k is achieved. Using this method, the safety of the spring loaded in all three selected working regimes is determined. Let us consider for simplicity a straight limit curve of the Haigh diagram, i.e. the equation with exponent ($s = 1$). In this case, the equation holds for this curve in a sectional form:

$$\frac{\tau_A}{\tau_C^*} + \frac{\tau_M}{\tau_f} = 1 \tag{5}$$

1. Regime with a constant mean tension, τ_m = const. (Fig. 6)

The working amplitude of the tension in the spring, $\tau_M = \tau_m$, is proportional to the working power; therefore, it holds for the limit amplitude (at point **M**) that $\tau_A = k_1 \times \tau_a$, while the mean tension remains the same, $\tau_M = \tau_m$. After substituting into the equation of the limit curve (5), we express safety k_1 as:

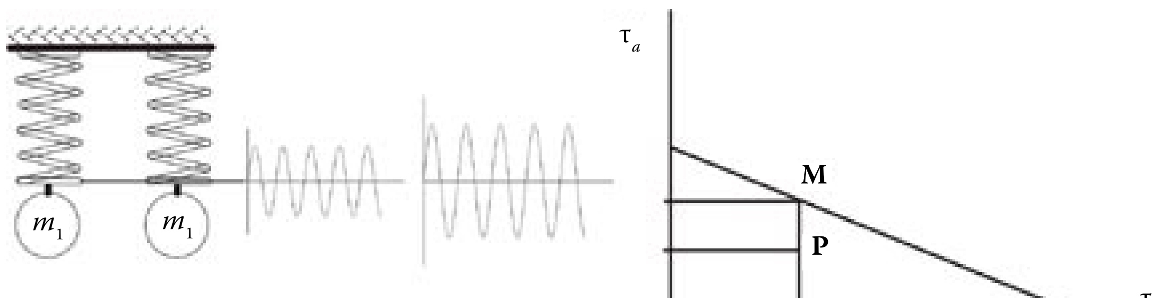


Fig. 6. Regime with a constant mean tension τ_m = const.

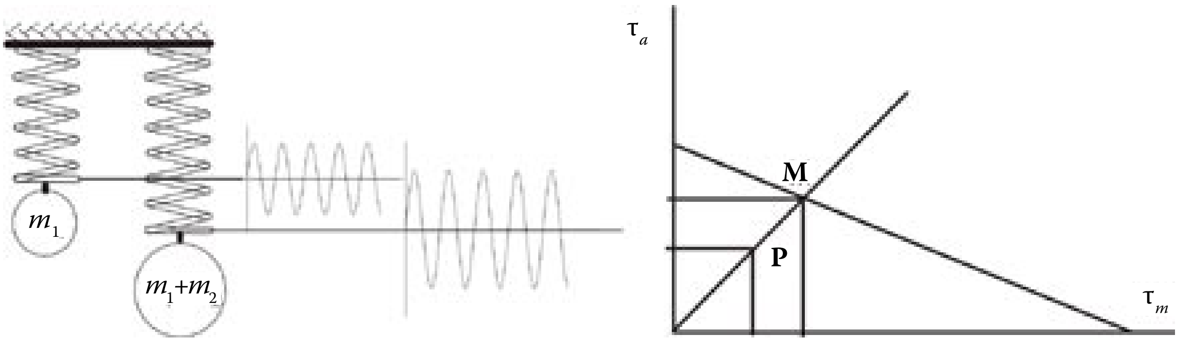


Fig. 7. Regime of proportional loading in components τ_a, τ_m

$$k_1 = \frac{\tau_C^*}{\tau_a}\left(1 - \frac{\tau_m}{\tau_f}\right) \tag{6}$$

2. Regime of the proportional loading in components τ_a, τ_m (Fig. 7)

Both components change proportionally to parameter λ, so that it holds on the limit curve that, $\tau_A = k_2 \times \tau_a$, $\tau_M = k_2 \times \tau_m$.

After substituting into Eq. (5), we obtain:

$$k_2 = \frac{\tau_C^* \times \tau_f}{\tau_a \times \tau_f + \tau_m \times \tau_C^*} \tag{7}$$

3. Regime with a constant low tension τ_d = const. (with static preloading, Fig. 8)

In the Haigh diagram, this case can be represented by a straight line parallel to the repeated loading cycle (i.e. subtending an angle of 45°), because the loading line indicates the value of the bottom tension with a constant magnitude as an intersection on both axis, τ_d = const.

The equation for this regime has the following form:

$\frac{\tau_A}{-\tau_d} + \frac{\tau_M}{\tau_d} = 1$, with $\tau_d = \tau_m - \tau_a$.

After substitution, we obtain $\tau_M = \tau_A + \tau_m - \tau_a$. Because the amplitude of the tension changes proportionately to parameter λ during the growth of the amplitude of the loading force, it holds again that $\tau_A = k_3 \times \tau_a$ on the limit curve. After substituting into the equation of the limit curve of the Haigh diagram (5), the required safety k_3 is obtained as

$$k_3 = \frac{\tau_C^*\left(\tau_f + \tau_a - \tau_m\right)}{\tau_a \times \tau_f + \tau_a \times \tau_C^*} \tag{8}$$

COMPARISON OF REGIMES AND CONCLUSION

A specific example of the calculation of safety enables the mutual comparison of the regimes. Let us consider a fatigue limit of a specific spring with τ_C^* = 300 MPa. Let the actual breaking resistance be τ_f = 1,000 MPa. Let the basic amplitude of the working tension at a "unit force" be τ_a = 50 MPa. We expect to create a preloading of $\tau_d = 5\tau_a$ in case No. 3. For the increasing values of parameter λ, the results of the safety changes are plotted in the

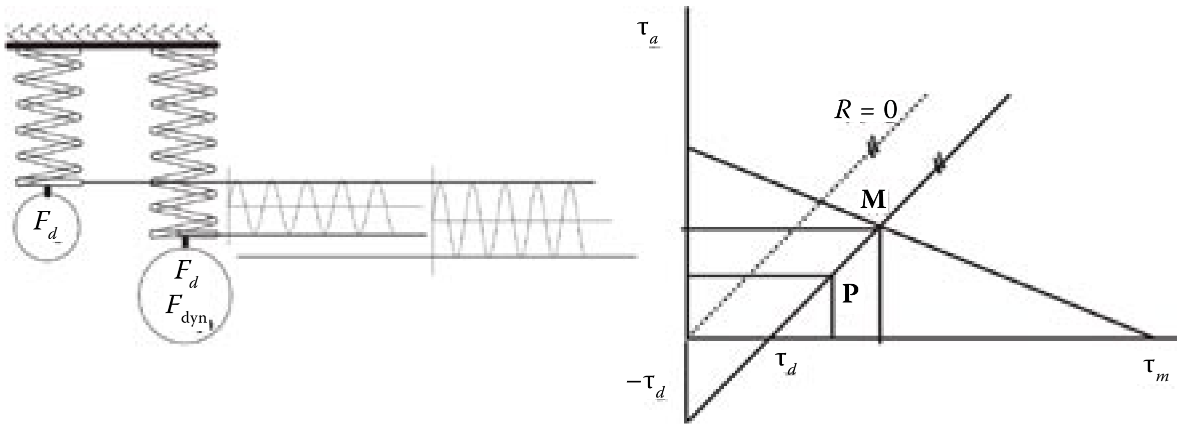


Fig. 8. Regime with a constant bottom tension, τ_d = const.

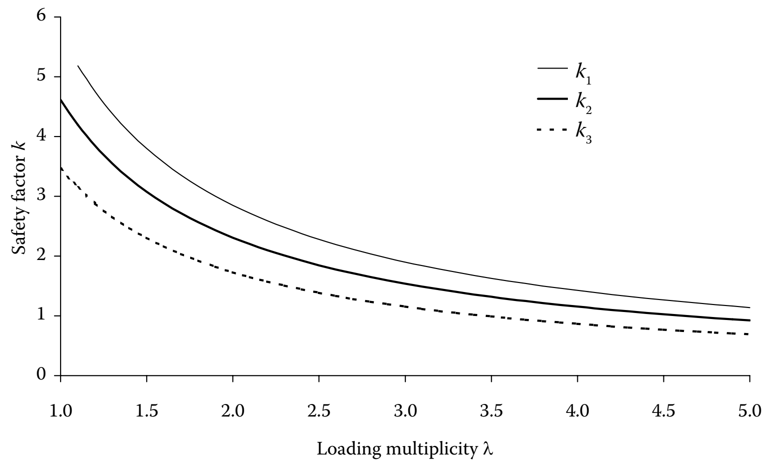


Fig. 9. Comparison of the results of the calculation of safety for three alternatives of loading springs

graph in Fig. 9. It is therefore obvious that in the region of the working loads, regime No. 1 leads to the highest safety. Conversely, the regime with preloading (No. 3) provides the lowest safety at the working level; it is therefore necessary during the design work to pay attention to the correct category of the working regime of springs according to the specific actual operation.

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Režimy zatížení a dimenzování na únavu válcových vinutých pružin

ABSTRAKT: Pružiny pracují často v režimu kmitavých zatěžovacích sil. Protože během své životnosti mohou absolvovat desítky, stovky až miliony zatěžovacích kmitů, hrozí u nich nebezpečí vzniku únavy materiálu, která může způsobit jejich porušení. Článek pojednává o faktoru koncentrace napětí na vnitřním poloměru válcových pružin a určení velikosti této koncentrace pomocí MKP. Výsledky těchto výpočtů pak byly srovnány s korekčními funkcemi publikovanými v literatuře. Určení bezpečnosti pružin s ohledem na mez únavy bylo provedeno pro tři typické zatěžovací režimy, které jsou znázorněny v Haighově diagramu. Z výsledků vyplývá, že nejmenší bezpečnost získáme pro případ se statickým předpětím pružiny.

Klíčová slova: pružiny; únava materiálu; zatěžovací režimy; bezpečný únavový život

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