Structural dynamic modification of circular disc using pre-stressed fields

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ABSTRACT: Dynamical properties of circular discs are investigated in this paper. One of the techniques of the disc modifications to achieve the required dynamic properties is to initiate pre-stress in disc plane. To obtain appropriate in-plane stress either roll-tensioning of disc surface or volume transformation of disc segment can be used. The role of in-plane stresses is assessed from the change in natural frequencies and modal shapes. The natural frequency characteristics for various rolling position and various rolling depth of the annulus are obtained by modal analysis using Finite Element Method (FEM).

Keywords: vibration; circular disc; in-plane stress; natural frequency; finite element method

The circular discs are structural elements widely used in the structural and processing applications. One of the most used geometric shape for processing and cutting operation material is thin circular disc – circular saw blade. Dynamical properties of circular saw blade are investigated in this paper. Circular saw blades are widely used for cutting and forming metal and non-metal materials. The quality of the cutting is mostly influenced by dynamical behaviour of circular saw blade in cutting process. The large transversal displacements of circular saw blade, induced by rotation of circular saw blade and external effects, occur during cutting process. In many cases, the loss of dynamical stability and arising of resonance state are caused by the reduction of natural frequencies of the disc due to the thermal compressive stress induced by cutting heat in the peripheral region. In dependence on cutting conditions, the temperature of cutting in the peripheral region increases up to 500°C. It is necessary to eliminate these inconvenient effects by design or technological treatments.

One of the techniques of the disc modifications to achieve the required dynamic properties is to initiate pre-stress (Markuš, Nánási 1981) in disc plane. This is concerned primarily with "tuning" of dynamical properties of vibrating circular discs by technological treatments inducing the residual in-plane stresses. It is possible to obtain the disc in-plane stress either by roll-tensioning of disc surface or by volume transformation of disc segment. In the roll-tensioning process, the disc is compressed within a certain annular contact zone between two opposing rollers. Plastic deformations in the contact zone of circular saw blade result residual stresses in whole disc plane. The effects of residual stresses induced by roll-tensioning on dynamical properties are analysed. The natural frequency characteristics for various rolling position and various rolling depth of the annulus are obtained by modal analysis using Finite Element Method (FEM). The role of residual stresses obtained by rolling is assessed from the changes in natural frequencies and modal shapes.

FORMULATION OF THE PROBLEM

The circular saw blade, which is analysed, has the shape of circular disc. In the following, we consider an isotropic homogeneous circular disc of an outer diameter D_0, inner diameter d_0 and thickness h (Fig. 1). The diameter d_f specifies a circle where the disc is clamped by flanges. The plastically deformed

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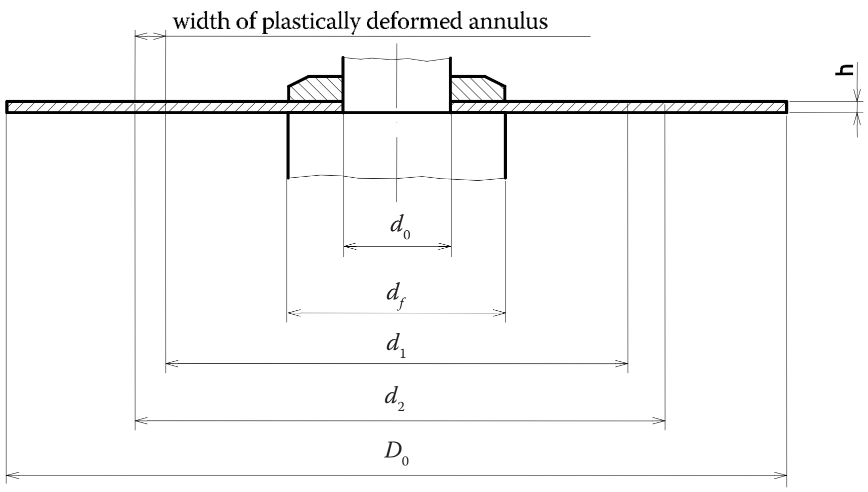

Fig. 1. The scheme of circular disc with plastically deformed annulus

zone of is determined by inner diameter d_1 and outer diameter d_2.

The fundamental considerations and derivation of equations of motion are based on Kirchhoff's assumptions. The assumptions are valid only for thin circular disc. The field of displacements in the cylindrical coordinates r, φ, z, using Kirchhoff plate theory, can be written as

$$u_r(r, \varphi, z, t) = u(r, \varphi) - z \frac{\partial w(r, \varphi, t)}{\partial r}$$

$$v_\varphi(r, \varphi, z, t) = v(r, \varphi) - \frac{z}{r} \frac{\partial w(r, \varphi, t)}{\partial \varphi} \tag{1}$$

$$w_z(r, \varphi, z, t) \cong w(r, \varphi)$$

where: $u(r, \varphi)$, $v(r, \varphi)$, $w(r, \varphi)$ – displacements of point laying on neutral plane of the circular disc in coordinate directions.

Generally, the stress-strain relations under consideration of initial stresses and initial strains are given by

$$\boldsymbol{\sigma} = \mathbf{D}(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}_0) + \boldsymbol{\sigma}_0 \tag{2}$$

where: $\boldsymbol{\sigma}$, $\boldsymbol{\varepsilon}$ – stress and strain vector,
$\boldsymbol{\sigma}_0$, $\boldsymbol{\varepsilon}_0$ – initial stress and initial strain vector,
$\mathbf{D}$ – elasticity matrix.

Using the finite element formulation (Zienkiewicz, Taylor 2000), the equation of motion for a free vibration of in-plane stressed disc is described by expression

$$\mathbf{M}\ddot{\mathbf{u}} + (\mathbf{K} + \mathbf{K}_\sigma)\,\mathbf{u} = \mathbf{0} \tag{3}$$

where: $\mathbf{M}$ – mass matrix,
$\mathbf{K}$ – tiffness matrix,
$\mathbf{K}_\sigma$ – stiffness matrix resulting from stress distribution induced by rolling,
$\ddot{\mathbf{u}}$, $\mathbf{u}$ – vector of nodal accelerations and vector of nodal displacements, respectively.

We note, that the mass distribution of circular disc after rolling is not changed, but the bending stiffness is considerably changed.

Equations (3) can be transformed to modal coordinates using the transformation equations

$$\mathbf{u}(t) = \boldsymbol{\varphi} q(t) \tag{4}$$

where: $\boldsymbol{\varphi}$ – modal vector,
$q(t)$ – normal coordinate of the system.

After applying the above transformation, the equation of motion (3) can be used to determination of the natural angular frequencies and mode shapes of the circular disc with roll-tensioning induced residual stress distribution. We obtain the following eigenvalue problem

$$(\mathbf{K} + \mathbf{K}_\sigma - \omega_i^2\mathbf{M})\,\boldsymbol{\varphi}_i = \mathbf{0} \tag{5}$$

where: $\omega_i = \sqrt{\dfrac{\boldsymbol{\varphi}_i^T(\mathbf{K} + \mathbf{K}_\sigma)\,\boldsymbol{\varphi}_i}{\boldsymbol{\varphi}_i^T\mathbf{M}\boldsymbol{\varphi}_i}}$ – natural angular frequency,
$\boldsymbol{\varphi}_i$ – eigenvector describing i-th modal shape of the circular disc.

In order to calculate the modification of disc stiffness $\mathbf{K}_\sigma$ after rolling, we must know the residual stress distribution in a disc plane. To determine the residual stress distribution, the method of thermal stress loading is used (Kuratani, Yano 2000). The thermal expansion induces a stress distribution, which is analogous to the stress distribution initiated by rolling. The dependence between temperature and depth of roll-tensioning is approximately described by equation

$$\Delta T \approx \frac{\mu}{h\alpha}\,\Delta z \tag{6}$$

where: μ – Poisson number,
α – temperature expansion coefficient,
h – disc thickness,
Δz – depth of roll-tensioning.

Table 1. Material properties of circular disc

Young modulus E (GPa)	Poisson number μ (–)	Density ρ (kg/m^3)
210.0	0.3	7,800.0

The matrices **M, K** and additional matrix $\mathbf{K}_\sigma$, which follow from stress distribution arising from rolling (in this model analogy with thermal expansion is used), are calculated automatically by ANSYS (ANSYS 2001). Solution processes for determination of natural angular frequencies and modal shapes are realised by ANSYS.

The natural angular frequencies and modal shapes of circular disc with residual stress distribution for different position, depth and width of roll-tensioning can be determined from equation (5). To obtain appropriate dynamical properties of circular disc, it is necessary to determine the natural frequency curves for the various positions of mean radius of the roll-tensioning annulus $R = (d_2 + d_1)/4$.

NUMERICAL EXAMPLE AND CONSIDERATIONS

We consider a circular disc (Fig. 1) of the outer diameter D_0 = 240 mm, inner diameter d_0 = 30 mm, thickness h = 1.8 mm and flange diameter is d_f = 50 mm. The width of plastically deformed annulus is assumed as 10 mm. This width is selected arbitrarily and it is considered as a representative value for planned experimental verification of investigated phenomenon. The rest of input data used for following numerical analysis of circular disc are introduced in Table 1. The analysed disc is assumed to be perfectly fixed in region $r \le r_f$. The outer edge of circular disc is free.

The solution of the eigenvalue problem (5) for free vibration of circular disc (Fig. 1) consists of the natural frequencies and the corresponding mode shapes. The individual mode shapes are represented by nodal lines and nodal circles and each mode shape is described by number of nodal circles/number of nodal lines. Nodal circles and nodal lines are sets of points of disc with zero deflection. The first four mode shapes of circular disc are plotted on Fig. 2 (black areas are parts of disc with zero deflections).

The effect of the mean radius of roll-tensioning annulus R on natural frequencies of circular disc is investigated. Fig. 3 shows the influence of rolling radius and depth of rolling on the natural frequencies for mode shapes 0/1, 0/0, 0/2, 0/3 (number

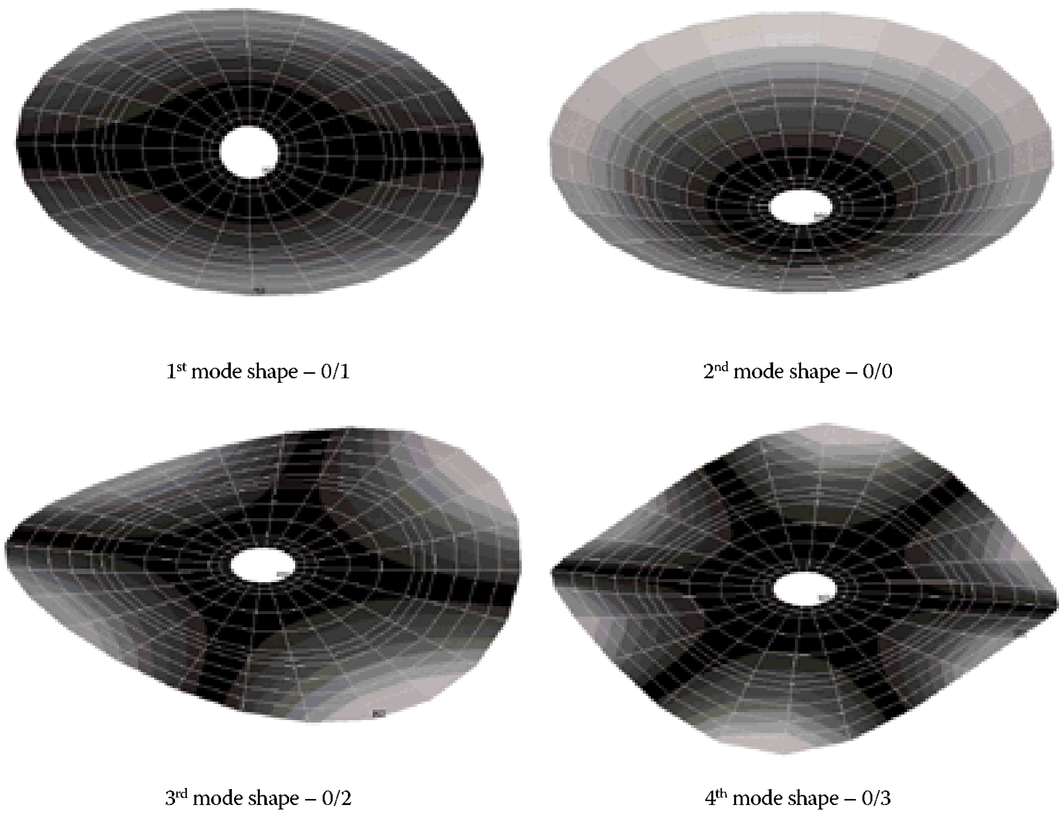

Fig. 2. Mode shapes of circular disc

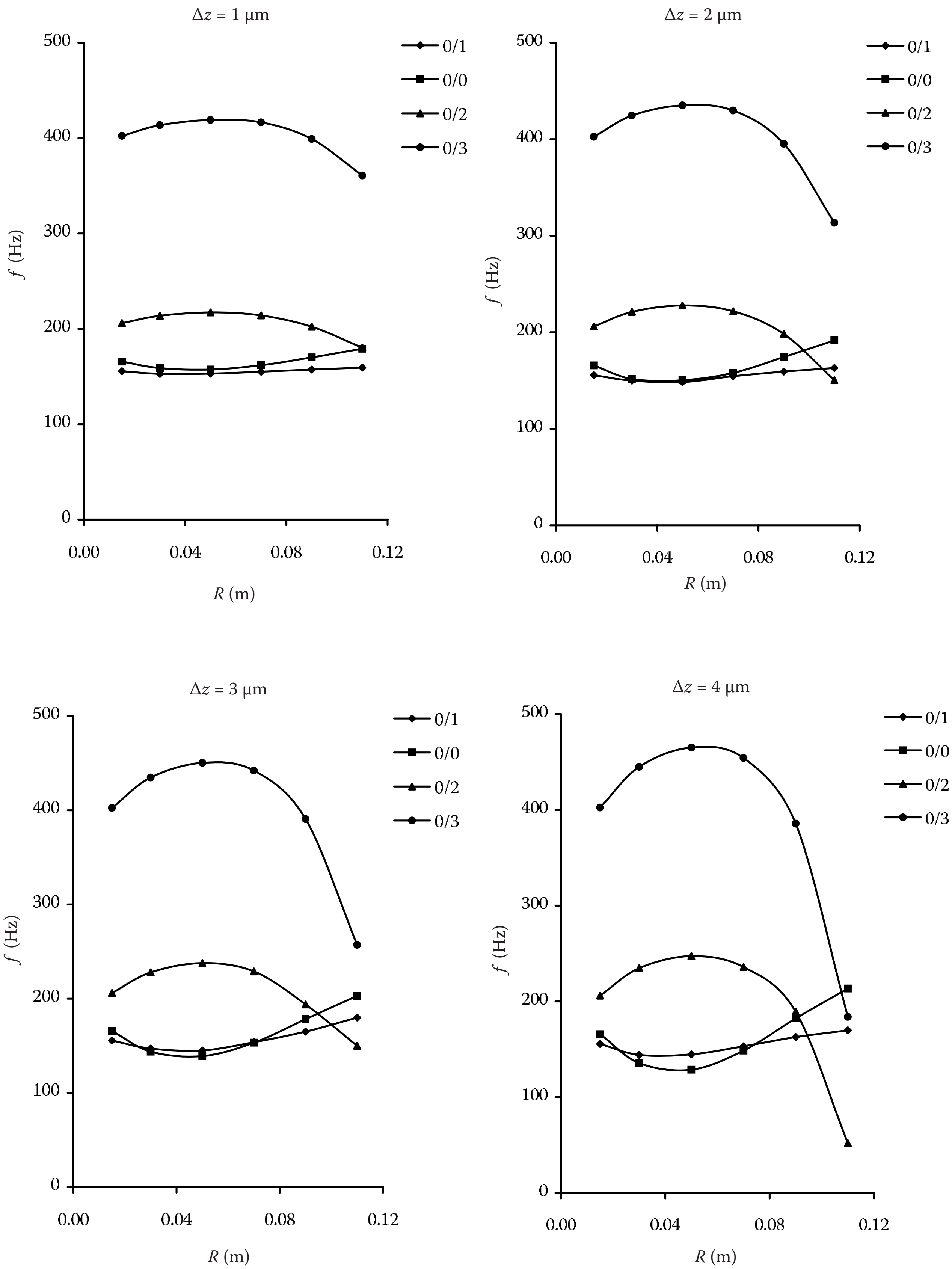

Fig. 3. Effect of rolling depth Δz and mean rolling radius R on the first four natural frequencies

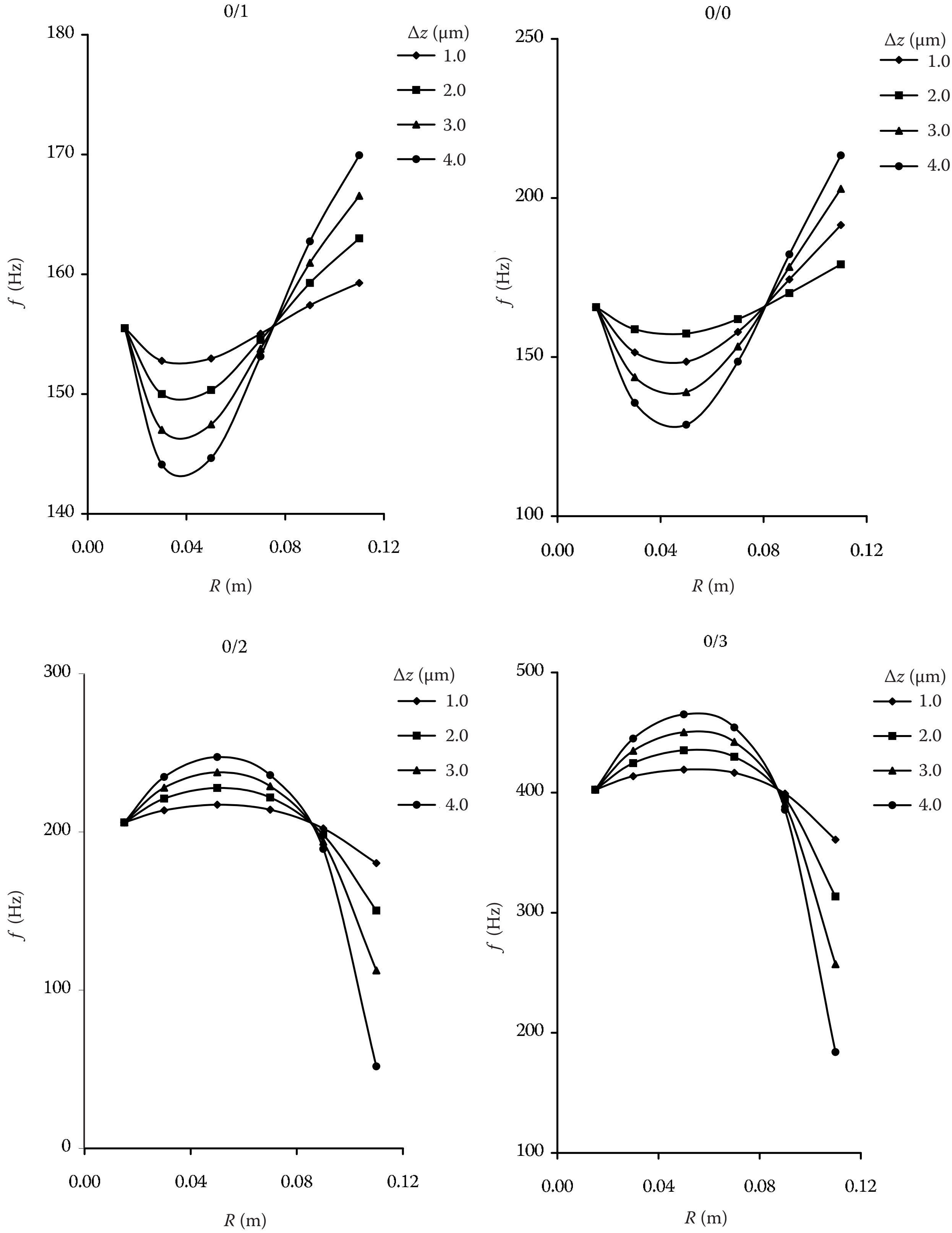

Fig. 4. Influence of rolling radius R and rolling depth Δz on the individual natural frequencies

of nodal circles/number of nodal lines) calculated by FEM. The mean radius of rolling R varies from 30 mm to 110 mm and depths of rolling are 0.1 μm, 0.2 μm, 0.3 μm and 0.4 μm. The natural frequencies of circular disc before rolling are marked by R = 15 mm. The tendency of frequency curves for modal shapes 0/1 and 0/0 differs from frequency curves for modal shapes 0/2 and 0/3. The natural frequencies of the modal shapes 0/2 and 0/3 increase with R until the maximum values near R = 60 mm are reached; then they decrease. Contrary to this, the natural frequencies of the modal shapes 0/1 and 0/0 decrease with R and for R = 50 mm reach the minimum; then they increase.

The effect of thermal compressive stress induced by cutting heat in the peripheral region can be seen for R = 0.11 m, where natural frequencies of the modal shapes 0/2 and 0/3 for different depths of rolling are lower then natural frequencies of the modal shapes 0/1 and 0/0. For this case, the resonance of circular disc arises for lower frequencies. The reason why natural frequencies are varied due to rolling is that stiffness characteristics within the disc are locally changed by stress distribution. The influence of mean radius R and depth Δz of plastically deformed annulus on the natural frequencies for individual modal shapes is shown on Fig. 4.

CONCLUSIONS

The theoretical formulation and mathematical model for analysis of dynamical properties of circular disc with residual stress distribution are presented. Finite element analysis for estimating the natural frequencies was used. For certain mean rolling radius R the natural frequencies of modal shapes 0/2 and 0/3 become smaller than those before rolling and the desired effect of tension stresses cannot be achieved. Therefore, the appropriate rolling position is necessary to be determined from natural frequency characteristics calculated for various mean rolling radius. This method of structural modification can be used to solve the various design problems of mechanical systems and it is very effective in changing the dynamic properties of similar structural elements, see for example Bodnicki (2000).

References

ANSYS, Release 5. 7. 2001. SAS IP, Inc., Canonsburg, USA.

BODNICKI M., 2000. Problems of design and application of rotary torque meters for measurement of a small torque. Journal of Theoretical and Applied Mechanics, *38*: 499–522.

KURATANI F., YANO S., 2000. Vibration Analysis of a Circular Disc Tensioned by Rolling Using Finite Element Method. Archive of Applied Mechanics, *70*: 279–288.

MARKUŠ S., NÁNÁSI T., 1981. Significance of In-plane Inertia Forces in the Vibration Analysis of Three-layered Circular Plates. Journal of Sound and Vibration, *76*: 421–441.

ZIENKIEWICZ O.C., TAYLOR R.L., 2000. The Finite Element Method, Volume 1–3. Oxford, Butherword-Heinemann: 1423.

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Štrukturálna dynamická modifikácia kruhového kotúča pomocou predpätých oblastí

ABSTRAKT: V článku sú skúmané dynamické vlastnosti kruhových kotúčov. Jednou z možností, ako dosiahnuť požadované dynamické vlastnosti, je modifikovať kotúč vytvorením predpätých oblastí v rovine kotúča. Vhodné predpätie môže byť vytvorené pomocou valcovania povrchu kotúča alebo vhodnou zmenou objemu časti kotúča. Vplyv napätí v rovine kotúča je hodnotený na základe zmeny vlastných frekvencií a vlastných tvarov kotúča. Závislosti vlastných frekvencií pre rôzne polohy valcovania a rôzne hĺbky valcovania prstenca sú určené pomocou modálnej analýzy použitím metódy konečných prvkov (MKP).

Kľúčové slová: kmitanie; kruhový kotúč; predpätie; vlastná frekvencia; metóda konečných prvkov

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