Cylindrical screw gearing with parallel axes of rotation

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Abstract: In the first part the article describes the theory of cylindrical screw gearing and in second part it states an alternative solution of screw gearing with parallel axes of rotation. There are given results of relating correctness of gear meshing and its manufacturing as theoretical problem. Authors present possibilities of screw gearing with parallel axes of rotation in area of agricultural machine application too.

Keywords: cylindrical screw gearing; correct gear meshing; gearing; agricultural machine

The screw gearings serve for creation of kinematical and force relationship between skew shafts. Its name is derived from motion of meshing elements rotating along fixed axes o_1, o_2, with rotational velocities ω_1, ω_2, thereby creating a relative screw motion with sliding velocity $v_k = v_{1t} - v_{2t}$ around instantaneous screw axis o_{12} (Figure 1). The gear ratio i of the rolling of hyperboloids is determined by their rolling component which give this relationship:

$$i = \frac{\omega_1 = r_2 \cos\vartheta_1}{\omega_2 = r_1 \cos\vartheta_2} \qquad (1)$$

Thus an actual gear ratio i is attainable by a suitable combination of length and angle parameters (Litvin & Fuentes 2004).

MATERIAL AND METHODS

Theoretical foundation of screw gearing is the hyperboloid gearing. One-part rotation hyperboloids with screw shafts o_1, o_2 are its axoids. Axoids originate by straight line rotation o_{12} along axes o_1, o_2 (Figure 2).

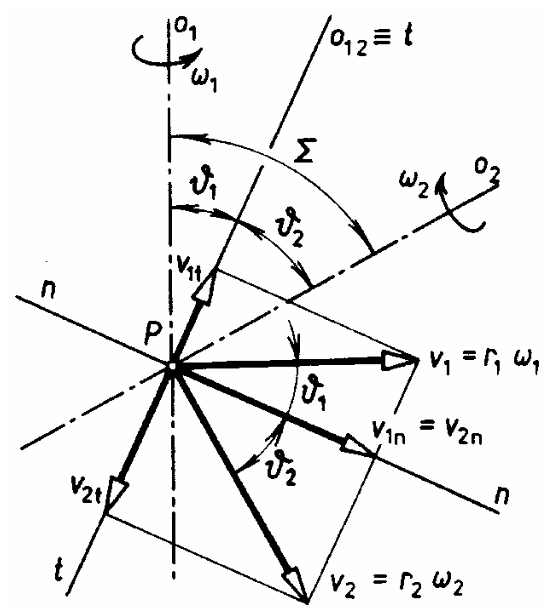

Figure 1. The velocities of meshing elements

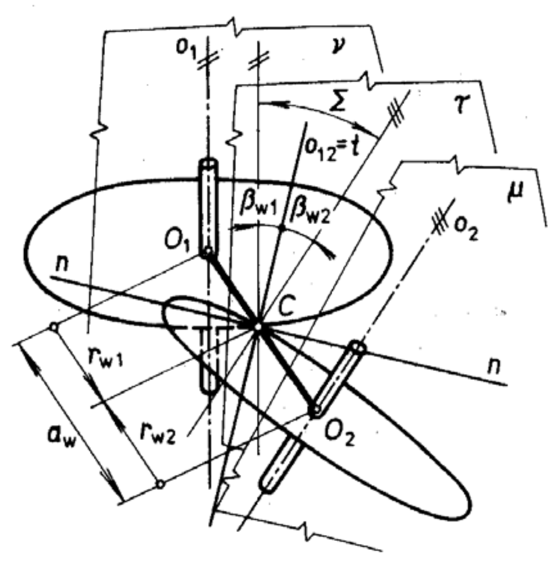

Figure 2. The contact of screw cylinders

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Velocity conditions in the screw gearing are evident from Figure 1. Contact hobbing condition is fulfilled if $v_{n1} = v_{n2} = v_n$. For sliding velocity in direction of o_{12} is valid the following equation:

$$v_k = v_{1t} - v_{2t} = r_1\omega_1 \sin(\vartheta_1) - r_2\omega_2 \sin(\vartheta_2)$$

Because v_{1t} and v_{2t} have opposite directions the afterwards equation is valid:

$$v_k = r_1\omega_1\sin(\vartheta_1) + r_2\omega_2\sin(\vartheta_2) \tag{2}$$

and for rotational velocity magnitude in axes direction o_{12} (Málik *et al.* 2003) is valid:

$$\omega_{1,2} = \omega_1\cos(\vartheta_1) + \omega_2\cos(\vartheta_2) \tag{3}$$

Further it is valid:

$$\Sigma = \vartheta_1 + \vartheta_2,\ a = r_1 + r_2 \tag{4}$$

From Eq. (1) up to Eq. (4) afterwards it results:

$$tg(\vartheta_1) = \frac{\sin(\Sigma)}{i + \cos(\Sigma)},\quad tg(\vartheta_2) = \frac{i\sin(\Sigma)}{1 + i\cos(\Sigma)},$$
$$r_1 = a\,\frac{tg(\vartheta_1)}{tg(\vartheta_1) + tg(\vartheta_2)},\qquad r_2 = a\,\frac{tg(\vartheta_2)}{tg(\vartheta_1) + tg(\vartheta_2)} \tag{5}$$

Only ascertained parts of contact – delineated by a pair of cross-sections – are used in design of mechanical gears. Hyperboloid gear wheels are thus created by providing the surface of these frictional wheels with appropriate toothing. The teeth will satisfy the condition of mutual rolling and sliding on axoid surfaces in the direction of axis o_{12} only if angles of teeth declination β_1 and β_2 satisfy these conditions:

$$\beta_1 = \vartheta_1 \text{ and } \beta_2 = \vartheta_2 \tag{6}$$

then the contact between meshing sides of teeth is theoretically linear. Fulfilment of condition (1) is not however unconditionally necessary. A pair of hyperboloids created by rotation of axis o_{12} gradually around axes o_1 and o_2 can have also toothing with angles of declination $\beta_1 \neq \theta_1$ and $\beta_2 \neq \theta_2$, but on condition that $\beta_1 + \beta_2 = \Sigma$ must be valid. In this case the axis o_{12} will take a new position and for gear ratio of screw gearing, the equation is valid:

$$i' = \frac{r_2\cos\beta_2}{r_1\cos\beta_1} = \frac{\omega_1}{\omega_2} \tag{7}$$

then the contact between meshing sides of teeth is theoretically a point.

Hyperboloid gearing is practically not used for technological reasons, only as theoretical starting point for two real screw gearings:

– cylindrical screw gearing – starting point is the shortest transversal position of skew axes where the shape of hyperboloid bodies is substituted with simplified circular cylinders,

– conical screw gearing – starting point is on the most faraway positions from the most short transversal where the shape of hyperboloid bodies is substituted with simplified truncate cones.

These shape simplifications will cause that the original linear contact of hyperboloids turns in meshing cylinders and cones into the point contact. This disadvantage however applies in particular to cylindrical screw gearings which therefore are used mainly as kinematical gears or only for transfer of small powers. In conical screw gearings the more advantageous linear contact can be reached by modification of toothing with circularly curved teeth.

At substitution of the contact hyperboloids in their most narrow part which is determined by radiuses

Figure 3. The screw gearing with parallel axes of rotation

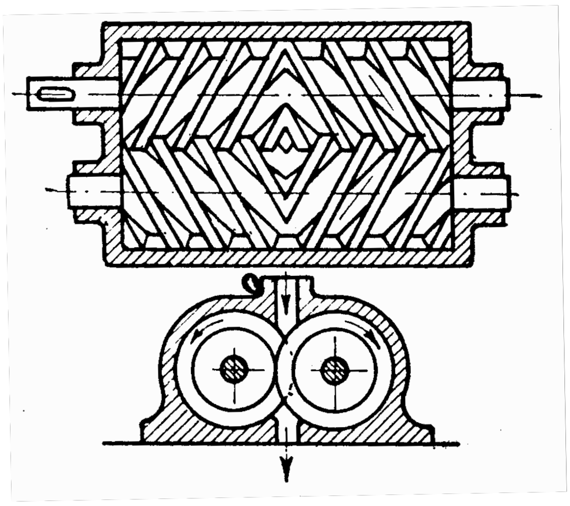

Figure 4. Helical gearing with parallel axes of rotation in application as pump

of cylinders in a point of minimum diameter of hyperboloid as mentioned above the original linear contact will turn into a point contact of screw cylinders. Therefore so-called screw cylinders with radii r_{w1}, r_{w2} (Figure 2) are geometric base of cylindrical screw gearing.

The centre distance a_w equals the shortest transversal of skew lines $\overline{O_1O_2}$, where the equation is valid:

$$r_{w1} + r_{w2} = \overline{O_1O_2} = a_w \quad (8)$$

Teeth of cylindrical screw wheels are winded in the helix shape so that the angles of lead of helixes accord with the angles β_{w1} and β_{w2} and the mesh relations in normal section meet the mesh relations according to the theory of plane toothing. These requirements will be fulfilled using two cylinder wheels with screw teeth as at cylinder gearings with the difference that the angles of lead of meshing wheels will be different and the directions of lead of helixes can be corresponding. Both wheels also are based on the same basic profile of toothing in common normal plane (Argyris *et al.* 1998). So two evolvent cylindrical wheels with screw teeth, which will create the cylindrical screw gearing, are generally determined by the following parameters of wheels: $z_{1,2}$, $x_{1,2}$, $\beta_{1,2}$, $b_{1,2}$ and parameters of basic profile in normal section which meet normalized values and are common for both wheels (are manufactured by the same tool).

RESULTS AND DISCUSSION

If we start from term of screw gearing so in comparison with definition of screw gearing as mentioned above, it is possible to come to definition of quite different gearing. Starting from basic law of toothing the gearing can be solved also by classic screw mesh. Thereby we can speak about screw gearing with parallel axes of rotation as shown on Figure 3 (it is a case of double thread or four threads).

There remains to solve questions relating to correctness of mesh of such gearing in particular as far as interference as well as manufacturing or shape of tooth. Such type of gear in the design with gear ratio $i = 1$ has already been used generally for a long time in applications as compressor or pump (Figure 4) (Arbon 1994). It has been used mainly as kinematical gear with gear ratio $i = 1$, however, in the design as shown in Figure 3, such type of screw gearing has not been used yet as power gear.

The target of solution was to consider whether it is possible to use such type of gearing also as power gear. However at geometric design of such gearing the questions arose partly of the design of shape of toothing – whether it would be evolvent shape of side of tooth or a choice of simple trapezoidal thread (Figure 5) and also of effective way of manufacturing with appropriate accuracy.

It appeared that such shape of tooth is simple but from point of view accessibility of manufacturing is not overly appropriate therefore we looked for another shape of tooth.

CONCLUSION

It appeared that in kinematical analysis of screw gearing with parallel axes of rotation that it is possible to provide manufacturing of such type of gearing by several ways with various accuracy

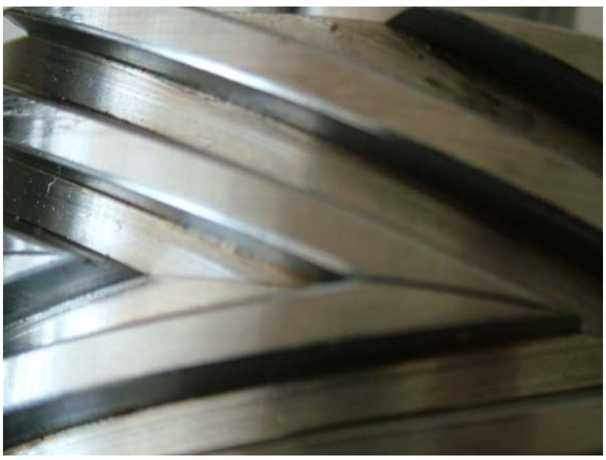

Figure 5. Trapezoidal shape of tooth

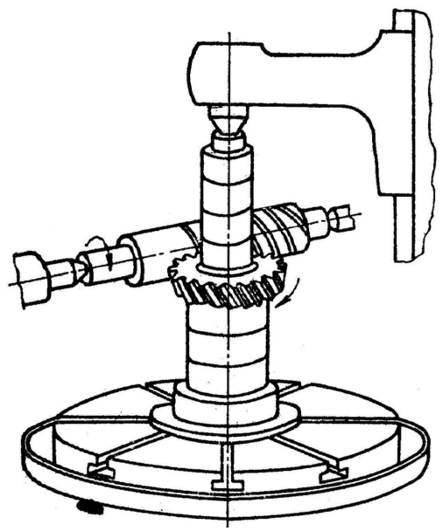

Figure 6. Manufacturing of crossed helical gear on hobbing cutter

Figure 7. Modified trapezoidal shape of tooth

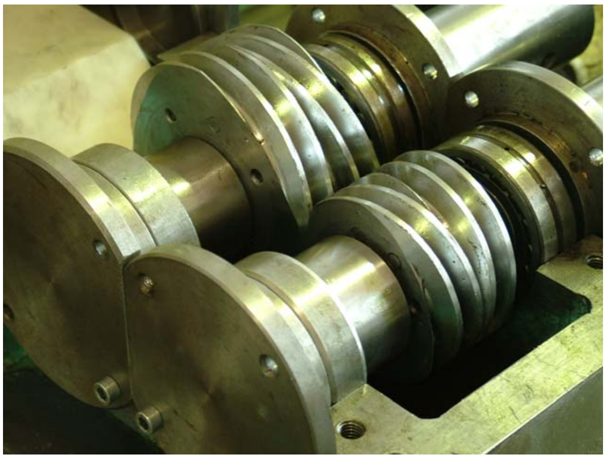

Figure 8. Helical gearing after test by loading 7^{th} stage

and various price (Nemčeková *et al.* 2006). The way by grinding appeared as impassable in spite of high accuracy for a reason of excessively high price. Therefore we looked for other options of its manufacturing. Here we ran up against restrictions relating to accessible machinery and qualified staff. The possibility of manufacturing of worm by the turning, by the milling with cutting disc or shank mill, by rounding machine as well as by using NC machines was being examined. Just accuracy, time difficulty, machinery or costs usually created the main barrier for the possibility of perspective application of this gear. Using the classic hobbing particularly used for manufacturing of cylindrical gears is very interesting method. Of course, crossed helical gear has helix angle beta which is too large that it could be manufactured by classic method of the milling. The worm can be manufactured on this mill but by reversed way (Figure 6). Semi-product for manufacturing of crossed helical gear (worm) will be fixed on arbor instead of the hobbing cutter. The tool will be placed on position where is usually fitted semi-product for manufacturing of crossed helical gear, on condition that the mill will have equipment for tangential milling. Diameter of manufactured crossed helical gear is limited by area for original tool of which diameter is given in basic technical parameters of each machine. The pinion shaped cutter which is usually used for hobbing shaping at manufacturing of evolvent toothed gears serves as a tool. The ratio between a tool and toothed gear is adjusted on machine tool. Apart from that the ratio change will be provided by differential gear depending on feed. On machine tool the cutting conditions – the cutting speed and feed which gives a size of cut – also will be correctly adjusted. According to the desired gear ratio of manufactured gearing it is necessary to correlate the number of teeth of the pinion shaped cutter and an option of the hobbing cutter. As an example we give that in case of gearing with gear ratio 2, one crossed helical gear can have a number of teeth (runs) 2 and another crossed helical gear can have 4. For this number of teeth it is necessary to have the pinion shaped cutter with a number of teeth in the first case divisible by two and the second case divisible by four so the pinion shaped cutter with a number of teeth 20, 24, 28, 32, etc. satisfies this case. After that it is necessary on the mill to adjust the gear ratio which equals quotient of a number of teeth of the pinion shaped cutter and a number of teeth (runs) of crossed helical gear. It follows that for example for a number of teeth of the pinion shaped cutter it is necessary to adjust transmission gear of machine tool in case of a number of teeth 2 to 16 and in case of a number of teeth 4 to 8. The lower limit of a number of teeth of transmission gear which can be adjusted is given in machine parameters; mostly it is possible to decrease this transmission gear by manufacturing of exchange wheels. As a result we had to correct some deviations from theoretical shape by additional operation of grinding finish. The result of solution is shape of tooth according to Figure 7. From the point of loading we subjected this shape of toothing on Nieman testing stand where this shape at 7^{th} loading stage in interaction with ecological lubricant showed lower temperatures of oil than in case of evolvent and also convex-concave toothing and surface of tooth do not show any changes (Figure 8). This solution would enamble the application of such toothings in drives of agricultural machinery and other technological equipment which are operated in conditions which demand the elimination of the damaging leakage of oil to environment (Bošanský *et al.* 2005).

References

ARBON I.M. (1994): The Design and Application of Rotary Twin-shaft Compressors in the Oil and Gas Process Industry. MEP, London.

ARGYRIS J. , LITVIN F.L., AOYONG PENG , STADTFELD H.J. (1998): Axes of meshing and their application in theory of gearing. Computer Methods in Applied Mechanics and Engineering, **163**: 293–310.

BOŠANSKÝ M., VEREŠ M., GADUŠ J. (2005): Possibilities of the Use of C-C Gearings in Agricurtural and Building Machines Working in Environments with Increased Environmental Hazard. Acta Technologica Agriculturae, **8**: 78–81.

LITVIN F.L., FUENTES L. (2004): Gear Geometry and Applied Theory. 2nd Ed., Cambridge University Press, Cambridge.

MÁLIK L., MEDVECKÝ Š. *et al.* (2003): Časti a mechanizmy strojov. EDIS, Žilinská univerzita, Žilina.

NEMČEKOVÁ M., BOŠANSKÝ M., VEREŠ M. (2006): Možnosti znižovania hlučnosti prevodoviek. Acta Mechanika Slovaka, **10** (4-B): 257–262.

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Abstrakt

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Kľúčové slová: valcové skrutkové súkolesie; korektný záber; ozubenie; poľnohospodársky stroj

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