The model of oscillating system with coil and its validation

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Abstract: This article demonstrates the solution of a dynamic system with a complex kinematical structure and rolling resistance in the Matlab-Simulink program. To validate the simulation, a physical model with an incremental sensor was established which allows us to measure the kinematical values while the system is in motion. The article also includes the simulation model block diagram and the calculated course of kinematical values. Numeric results were compared with the real model. The measurement proved good conformity in basic parameters such as the period time, amplitude decrease, stop time etc. Small deviations in the final phase may have been caused by fine unevenness of the plane during the experiment.

Keywords: dynamic systems; mathematical model; simulation; validation

Mathematical modelling and simulation of the dynamic systems have become an integral part of the machinery design and construction. To establish a functional and credible simulation model, it is necessary to create an alternative mechanical scheme consisting of preferably all features affecting the system behaviour, the creation of the motion equation system (i.e. mathematical model with the description of the active parts as well as passive resistances and mechanical structures), and the creation of the computerised simulation model ensuring the solution of the mathematical model for the system parameters and initial conditions given.

The mathematical model of the system is compiled by the equations of motion based on the Newton's second law, d'Alembert's principle, free body methods or second Lagrangian equation (Beer & Johnston 1988; Bedford & Fowler 2005). The equations of motions are systems of non-linear differential equations a suitable instrument for the simulation of the mathematical model is e.g. program Matlab-Simulink. Samples of the solution of different mechanical, hydraulic or thermodynamic systems are e.g. in publications (Jirků & Kočárník 2004, 2005; Jirků &Vondřich 2002; Kočárník & Jirků 2006).

The correctness of the methods and procedures used should be advisably to be validated by physical models scanning the kinematical values and by comparison of the simulation results with the values measured.

This article shows the solution of a system with complex kinematical structure and rolling resistance. The system is that of a weight and a coil rolling down the inclined plane (Figure 1). It is set in motion due to the weight of both system elements. Depending on the α fibre angle, the moment of the inner force S_1 (regarding the cylinder motion pole) changes its orientation. The system can thus perform muffled periodic motion.

METHODS OF MODELLING

The motion equations were derived from the free body method. Figure 1 shows the primary forces, the reactions in outer as well as inner structures and inertia forces, or moments. The rolling resistance is respected by displacing the reaction normal element between the cylinder and the plane by the rolling resistance arm ξ in the motion direction. The change of the arm position is respected by the sgn $\dot{x}$ function. The following motion equations are valid for each object:

Weight:

$$-S_2 - m_3\ddot{y} + m_3g = 0 \tag{1}$$

Pulley:

$$-S_1r_2 + S_2r_2 - I_2\ddot{\varphi}_2 = 0 \tag{2}$$

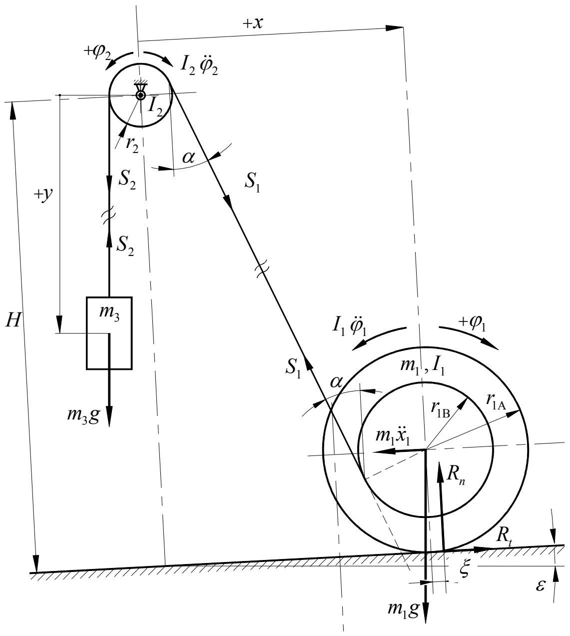

Figure 1. Mechanic scheme of the system

Coil on inclined plane:

$$R_t - m\ddot{x} - S_1 \sin\alpha - m_1 g \sin\varepsilon = 0$$

$$R_n - m_1 g \cos\varepsilon + S_1 \cos\alpha = 0 \tag{3}$$

$$S_1 r_{1B} - R_t r_{1A} - R_n \xi \operatorname{sgn} \dot{x} - I = 0$$

The equations can be completed with kinematical relations. The following relation applies to the rolling of the cylinder

$$\dot{x} = r_{1A}\omega_{rel} = r_{1A}\dot{\varphi}_1 \Rightarrow \ddot{\varphi}_1 = \frac{\ddot{x}}{r_{1A}} \tag{4}$$

The relation between the motion velocity of the coil $\dot{x}$ and the weight $\dot{y}$ can be derived from the projection of the P point resultant velocity $\vec{c}_P$ (given by the vector sum of drift and relative velocity $\vec{c}_P = \vec{c}_{Pdrift} + \vec{c}_{Prel}$) to the fibre direction, see Figure 2.

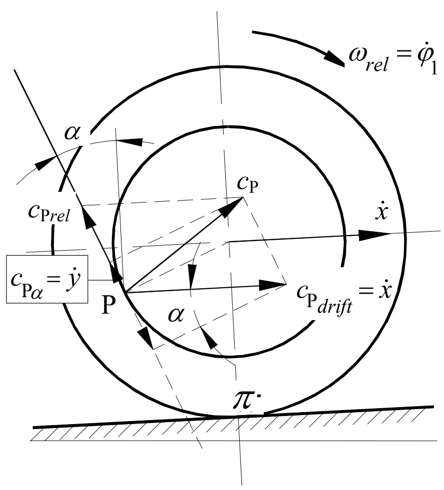

Figure 2. Kinematical structure

$$c_{Pdrift} = \dot{x},\ c_{Prel} = r_{1A}\omega_{rel} = \frac{r_{1B}}{r_{1A}}\dot{x} \Rightarrow c_{P\alpha} = \dot{y} =$$

$$= c_{Prel} - c_{Pdrift} \sin\alpha = \dot{x}\left(\frac{r_{1B}}{r_{1A}} - \sin\alpha\right) \tag{5}$$

After derivation of the last relation, we acquire

$$\ddot{y} = \ddot{x}\left(\frac{r_{1B}}{r_{1A}} - \sin\alpha\right) - \dot{x}\,\frac{d(\sin\alpha)}{dt} =$$

$$= \ddot{x}\left(\frac{r_{1B}}{r_{1A}} - \sin\alpha\right) - \dot{x}^2\,\frac{d(\sin\alpha)}{dx} \tag{6}$$

According to the figure geometry, the following relation applies for the angle α

$$\operatorname{tg}\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{x - (r_{1B} - r_2)\cos\alpha}{h - r_{1A} + (r_{1B} + r_2)\sin\alpha} \tag{7}$$

which can be adjusted to

$$\alpha(x) = \arcsin\left(\frac{x\sqrt{x^2 + (h - r_{1A})^2 - (r_{1B} + r_2)^2}}{x^2 + (h - r_{1A})^2} - \frac{(h - r_{1A})(r_{1B} + r_{1A})}{x^2 + (h - r_{1A})^2}\right) \tag{8}$$

After the elimination of the reactions and the inner forces from Eq. (1) to (3) and the establishment of derived kinematical Eq. (6), we obtain the following motion equation of the system

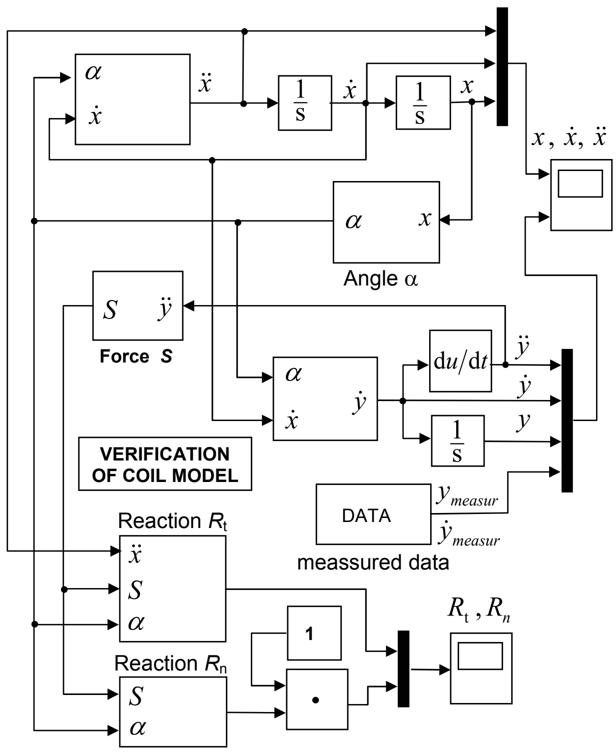

Figure 3. Simulation model scheme

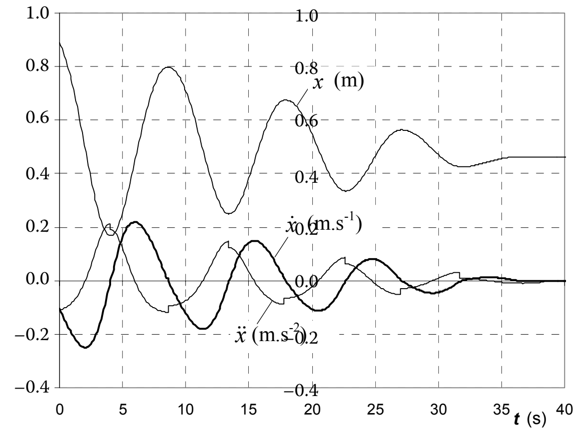

Figure 4. Coil distance, velocity and acceleration

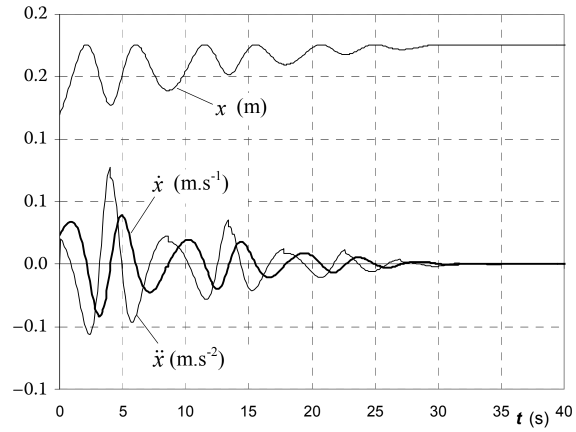

Figure 5. Weight distance, velocity and acceleration

$$\ddot{x} = \frac{\left[m_3 g + \dot{x}^2\left(m_3+\frac{I_2}{r_2^2}\right)\frac{\mathrm{d}(\sin\alpha)}{\mathrm{d}x}\right]\left(\frac{r_{1B}}{r_{1A}} - \sin\alpha + \frac{\xi \,\mathrm{sgn}\dot{x}}{r_{1A}}\cos\alpha\right) - m_1 g\left(\sin\varepsilon + \frac{\xi\,\mathrm{sgn}\dot{x}}{r_{1A}}\cos\varepsilon\right)}{\left(m_3+\frac{I_2}{r_2^2}\right)\left(\frac{r_{1B}}{r_{1A}}-\sin\alpha\right)\left(\frac{r_{1B}}{r_{1A}}-\sin\alpha+\frac{\xi\,\mathrm{sgn}\dot{x}}{r_{1A}}\cos\alpha\right)+\left(\mathrm{m}_1+\frac{\mathrm{I}_1}{r_{1A}^2}\right)} \quad (9)$$

With respect to the complexity of the relation for $\alpha(x)$, which is not a function of $\dot{x}$, the derivation of the sin α function can be performed numerically in a simulation model.

RESULTS

Figure 3 shows the model scheme in the Matlab-Simulink program. Figures 4 and 5 show time flows of kinematical values for the coil and weight motion.

The physical model for the validation of the numerical model is shown in Figure 6 and the detail of coil is in Figure 7. The photo of the model is presented in Figure 8.

The direct measurement of the coil position is technically difficult, so an indirect method was chosen which compares the weight position during the validation of the model characteristics. The x coil position must be converted to the y weight position in the numerical model using the Eq. (5) with subsequent integration. The weight position relates to the pulley displacement, which is scanned by an incremental sensor.

The determination of the initial conditions value for the numerical model is rather complicated, so the problem of the coil position measurement cannot be avoided. However, the problem can be bypassed so that the coil passage through the optical gate is scanned in the known position of $x_{opt} = x_{(t=0)}$. The

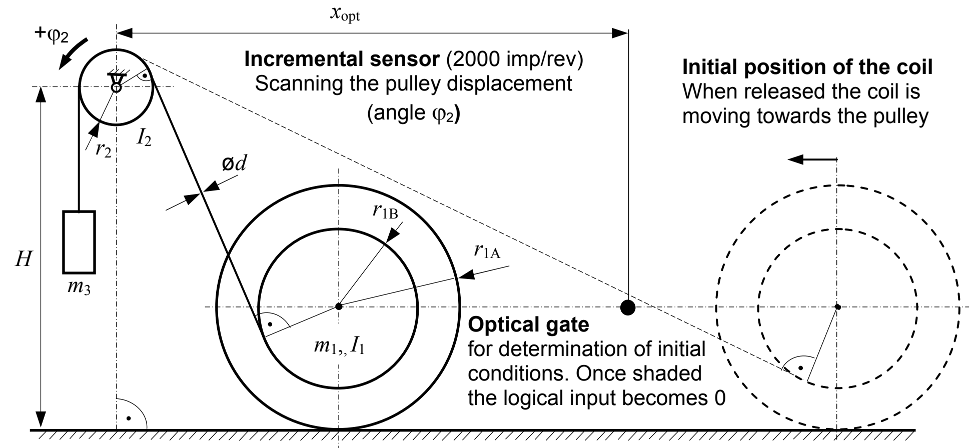

Figure 6. Physical model arrangement

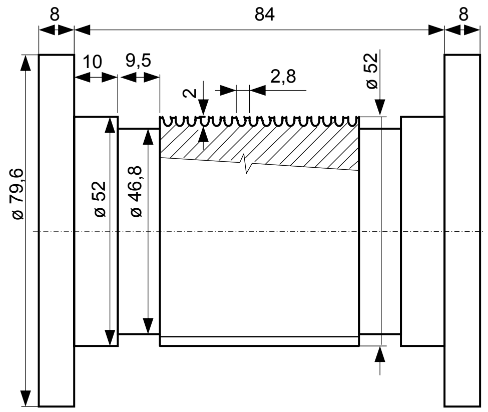

Figure 7. Detail of coil

coil velocity $\dot{x}_{(t\,=\,0)}$ can be determined in this position from the calculation resulting from Eq. (5). The required velocity $\dot{y}$ can be determined from the numerical derivation of the measured position y. The physical model parameters (weights, moments of inertia, etc.) are set by measuring and weighing, the unknown values for the ξ arm rolling resistance and ε plane inclination are determined from sequential optimisation so that the highest conformity of the compared courses is achieved (Figure 9).

CONCLUSION

The measuring demonstrated a good conformity in basic parameters, such as the period time, amplitude decrease, stop time etc. Small deviations in the final phase of the motion were most probably caused by fine unevenness of the working plane because the task is very sensitive to its inclination.

In the same way, the model was verified of a truck (driving moment M, respective supply voltage U of motor) with physical pendulum (Figure 9). The system has 2 degrees of freedom (case without slipping of the driving wheel) or 3 degrees of freedom (case with slipping of the driving wheel) the verification of accuracy of mathematical models is presented on a comparative physical model of this system with electronic control and scanning of kinematics quantities.

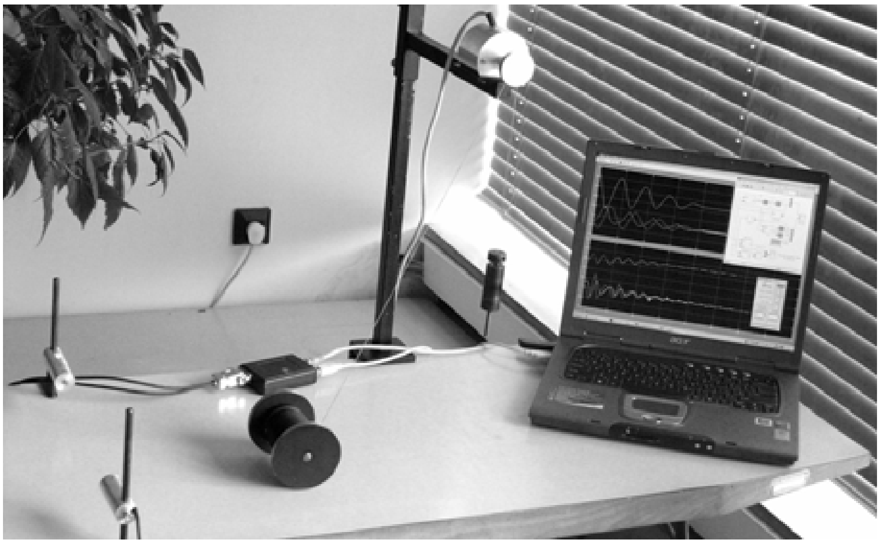

Figure 8. Photo of coil model

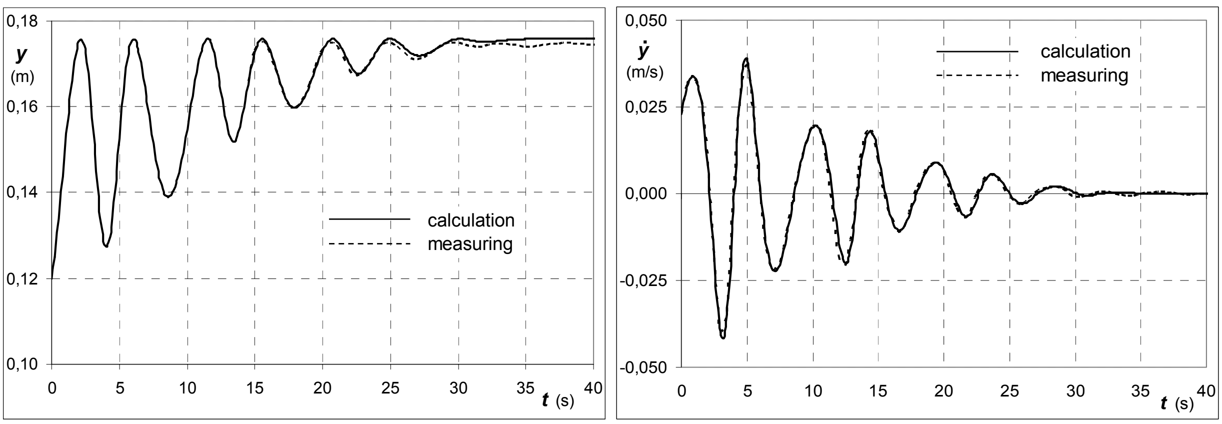

Figure 8. Calculated and measured courses of the distance y and velocity $\dot{y}$

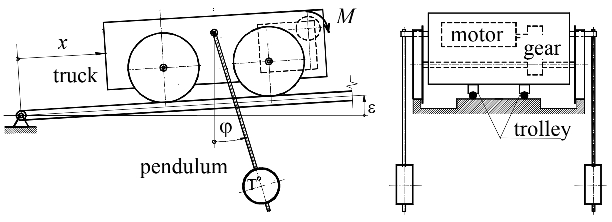

Figure 9. Model of the truck

The results of simulation shown in the experiment correspond to the exciting of the system by the moment of the motor drive (supplied with square wave voltage). The measured time-dependents of the truck position x, truck velocity $v = \dot{x}$, amplitude

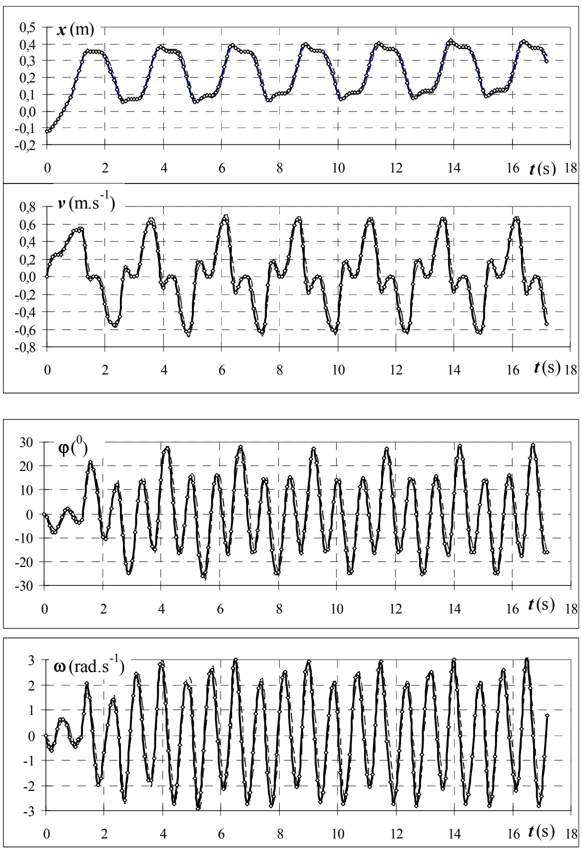

Figure 10. Comparison of measured and computed kinematical quantities for model of the truck

of pendulum φ and angular velocity $\omega = \dot{\varphi}$ with corresponding quantities determined by numeric solution of the mathematical model (dashed line) are compared in Figure 10. A detailed description of this problem is in Vodřich *et al.* (2001).

The results prove the correctness of the mathematical description, computer simulation, and adequate accuracy of the applied method identification of the system parameters.

The system described and checked in the article will probably not find direct use in practice. However, the philosophy of mathematical simulation mentioned and the experimental verification of dynamic systems are usable in different engineering spheres, e.g. in the research, proposal, and construction of agricultural machines.

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Abstrakt

Kočárník P., Jirků S. (2007): **Model kmitajícího systému s cívkou a jeho ověření.** Res. Agr. Eng., **53**: 182–187.

Článek je ukázkou řešení dynamické soustavy se složitou kinematickou vazbou a s odporem valení v programu Matlab-Simulink. Pro ověření simulace byl zkonstruován fyzikální model s inkrementálním čidlem, které umožňuje snímání a měření kinematických veličin za pohybu soustavy. V příspěvku je uvedeno blokové schéma simulačního modelu a vypočtené průběhy kinematických veličin. Numerické výsledky jsou porovnány s reálným modelem. Měření prokazuje dobrou shodu v základních parametrech, jako je doba periody, pokles amplitudy, doba zastavení apod. Malé odchylky v konečné fázi pohybu mohou být způsobeny drobnými nerovnostmi základové desky při experimentu. V závěru jsou stručně popsány i výsledky ověření simulace soustavy vozíku s kyvadlem.

Klíčová slova: dynamické soustavy; matematický model; simulace; ověření

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