Loading capacity determination of the wooden scarf joint

D. Herák[1], M. Müller[2], R. Chotěborský[2], O. Dajbych[1]

[1]*Department of Mechanical Engineering, Faculty of Engineering, Czech University of Life Sciences Prague, Prague, Czech Republic*
[2]*Department of Material Science and Manufacturing Technology, Faculty of Engineering, Czech University of Life Sciences Prague, Prague, Czech Republic*

Abstract: The paper describes the complete derivation of the theoretical bonded scarf joint loading capacity and the construction of the Mohr's circle for linear state of stress. Then the method of the experimental determination of the bonded joint loading capacity is explained in detail. A part of this paper deals also with the bonded joint real loading capacity determination.

Keywords: Mohr's circles; linear state of stress; bonding; scarf joint; wooden joint

Bonding is much more than art, it is science and it is exploited ever more. The choice and the use of a suitable adhesive is very important in productive and repair industry. A suitable choice saves costs and enhances the quality (Franklin International 1998).

Wood is one of raw materials which renews all the time. In contrast to metals it is a porous, non-homogenous, water-absorbing and volume labile material, whose base is cellulose and, according to the wood type, certain additional substances, as well. Wood possesses a number of excellent properties: elasticity, strength, toughness, mach inability etc. The selection of the correct wood type makes the use of wood possible in a great spectrum of human activities, from building industry to medicinal applications (Němec *et al.* 2005).

Generally, wood possesses positive mechanical properties. Mechanical properties of wood can be characterised as the ability to defy the deformation caused by outer forces. These properties are utilised when wood is used as the construction material.

Basic stress of wood

According to the stress type, we can distinguish the strength in the fibres direction – deformation appears as the body elongation. In the last stage, the rupture of the body tissue occurs. The ruptured part of a higher strength wood is fibriform, of a minor strength wood it is stairform or almost smooth. With the increasing moisture content the tensile strength across the fibres decreases up to the limit of the cell walls saturation (Lorenz 2002).

By the compressive force acting along the fibres deformation appears – the body shortens. The deformation character depends on the quality and structure of the wood. The density and the moisture content are important factors.

Anisotropy is one of the most important properties of wood (the mechanical properties are different in different directions).

Wood is a fibriform material and therefore it is much tougher in the longitudinal axis (in the fibres direction) than in the normal (transversal) direction. But the anisotropy of wood is three-dimensional. Therefore, we distinguish the longitudinal, radial, and tangential directions.

The values of individual mechanical properties of wood are different in various directions. The tensile strength of green wood is about five times lower than that of the same dry wood. Mechanical properties of wood, namely strength, are influenced by a number of factors, e.g. density, moisture content, knots, wood defects etc. (Pizzi 1994).

Supported by the Internal Grant Agency of Faculty of Engineering, Czech University of Life Science in Prague, Project No. 31140/1312/313104, CZU v Praze.

Bonding of wood

The bonding technology, which is very prospective even at present, can be used for the wood joining. It is used for bonding various materials. This technology can be used for solving otherwise difficult problems. For wood bonding, it is necessary to use quality and above all dry semi-products (ASTM 1994).

The advantages of the bonded wooden joints are:
– they do not reduce the bonded woods
– they are relatively strong

In wood bonding, the wood properties must be taken into account, namely the wood hardness and the structure of the bonded parts. The flushing of hard woods must be more thorough than of soft woods. Minute surface unevenness of soft wood can be suppressed by elevated clamping pressure of the assembled joint. On the contrary, using this practice on hard wood can lead to the adhesive being pressed out from the bonded joint (Morlier 1994).

The size of the bonded surface is the substantial factor affecting the bonded joint strength. When designing a bonded joint, the size of the bonded surface transmitting the load should be as large as possible. Therefore, various shapes of contact surfaces are used (scarf joints, indented joints). In bonding wood, four basic positions of the wood structure come into consideration. The optimum strength of a wooden joint is reached at parallel arrangement of annual shoots of the bonded surfaces. The butt joint is suitable for the wooden parts bonding, especially when a sufficient bonded surface is at disposal. In using butt joints, the scarf joints are advantageous (Müller 2006).

Before bonding the wood surface preparation is necessary:
– evenness preparation – the smoothly worked surface is the most suitable (the unevenness should be max. ± 0.2 mm). It can be reached e.g. by planing, milling, or grinding. After grinding, the dust removal is necessary. The working of hard wood must be more precise than that of soft wood. When bonding rough surfaces, it is necessary to spread the adhesive with a filler (Herák *et al.* 2005)
– resins, wax, and impregnation removal – mechanical, e.g. using brushes
– improvement of water content – we practice the temperature treatment of bonded parts

Every wood contains a specific quantity of water. We must eliminate the majority of water before the correct use of the wood. Wood loses or accumulates water to reach the state of equilibrium according to the storage and use conditions. This state of equilibrium depends on the air humidity and the temperature (Mukam & Mouongue 2003).

The contemporary offer of adhesives suitable for wood bonding enables a large selection of these products. It is necessary to choose the suitable product for the pertinent application. The comparison of prices is also important. The price often depends on the knowledge of the respective product.

The bonded wooden surfaces are conventionally larger. At bonding larger surfaces, the disperse types of adhesives are suitable. The advantage of these adhesive types is a simple use. The usable life of a majority of adhesives is sufficient for the easy and correct assembly of the bonded surfaces. The suitable moment of the bonded surfaces connection can be recognised according to the adhesive colour. After being applied on the bonded surface, the adhesive is non-transparent. After a certain time, it begins to lose slowly its coloring. In this phase, additional corrections are not recommended, because by now the coupling forms between the adhesive and the bonded parts.

Disperse solvent-type adhesives

The film-forming substance of the disperse adhesives is a very fine dispersion of polymers dispersed in water. After the water imbibition and evaporation, the sintering of polymeric particles in a coherent film occurs. The minimal film-forming temperature ranges to about 10°C–12°C. Compared with solvent-type adhesives, some advantages of the polymers dispersed in water are:
– low viscosity even at high dry basis contents of 50%–60%
– very low contents of inflammable and health-damaging organic solvents
– thinning ability by water and water washable aids
– workability at room temperature as well as one-component adhesives and the use without curing additives (Pizzi & Mitta 2003).

MATERIAL AND METHODS

Theoretical loading capacity of the wooden bonded scarf joints

For the determination of the loading capacity of the wooden bonded scarf joints, must be derived which describe the shear and normal stress distribution.

If we consider the section according to Figure 1, we determine the principal component of stress as

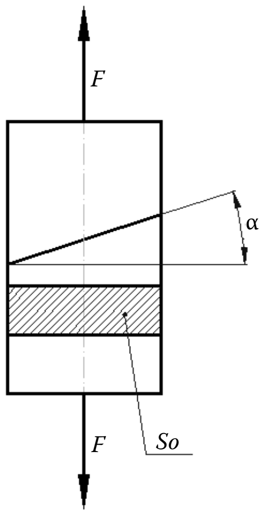

Figure 1. Diagram of the bonded scarf joint

$$\sigma_o = \frac{F}{S_o} \qquad (1)$$

where:

S_0 – cross section of the sample

If we resolve the force F into the normal and tangential components, we receive the following equations, which describe the individual force components (Figure 2).

$$N = F \times \cos\alpha \qquad (2)$$

$$T = F \times \sin\alpha \qquad (3)$$

We express the bonded surface according to the following Eq.

$$S = \frac{S_o}{\cos\alpha} \qquad (4)$$

If we introduce (3) and (4) into the basic Eq. of the shear stress, we obtain the Eq. which describes the relation between the shear stress and the cross section angle α (5) (Figure 3).

$$\tau_\alpha = \frac{T}{S} = \frac{F \times \sin\alpha}{\frac{S_o}{\cos\alpha}} = \frac{F}{S_o} \times \sin\alpha \times \cos\alpha = \frac{\sigma_o}{2} \times \sin 2\alpha \qquad (5)$$

If we introduce (2), (4) into the basic Eq. of the tensile stress, we obtain the Eq. which describes the relation between the normal stress and the cross section angle α (6) (Figure 3).

$$\sigma_\alpha = \frac{N}{S} = \frac{F \times \cos\alpha}{\frac{S_o}{\cos\alpha}} = \frac{F}{S_o} \times \cos^2\alpha = \frac{\sigma_o}{2} \times (1 + \sin 2\alpha) \qquad (6)$$

By graphical representation of the foregoing Eqs., we obtain the representation by means of the circle of the linear state of stress (3). According to its discoverer, this circle is also called the Mohr's circle of linear state of stress or uniaxial state of stress.

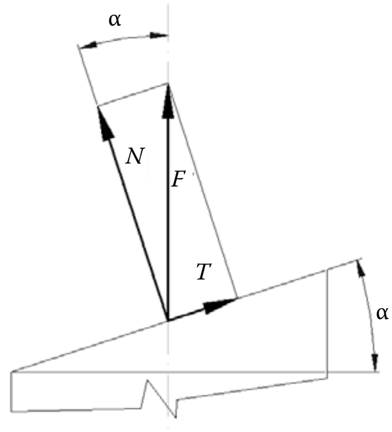

Figure 2. Forces diagram of the bonded scarf joint

Each adhesive has its failure strength in the normal direction (tensile strength) and in the tangential direction (shear strength). These data are determined from mechanical tests of the bonded joint (Ševčík 2005) described further.

If we plot the theoretical limiting tensile strength in the circle of stress, we can simply determine the minimal angle which defines the surface of the bonded joint optimum applicability.

$$\sigma_{mez} = \frac{\sigma_o}{2} \times (1 + \cos 2\alpha_{min}) \qquad (7)$$

$$\alpha_{min} = 2 \times \arccos\left(2 \times \frac{\sigma_{mez}}{\sigma_o} - 1\right) \qquad (8)$$

If we plot the theoretical limiting shear strength in the circle of stress, we can simply determine the maximal angle which defines the surface of the bonded joint optimum applicability.

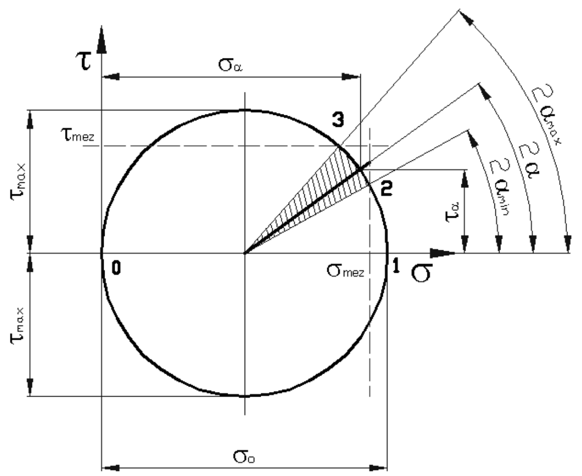

Figure 3. Mohr's circle of the linear state of stress of the bonded joint

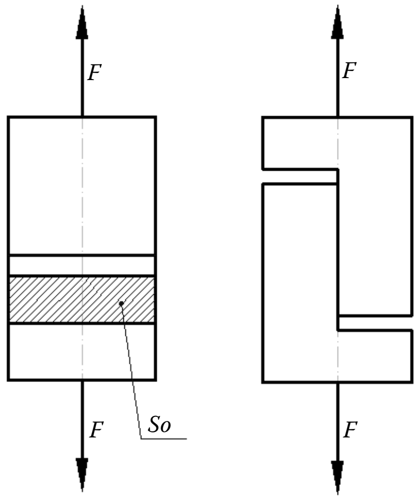

Figure 4. Diagrams of the bonded butt joint and the lap joint

$$\tau_{mez} = \frac{\sigma_o}{2} \times \sin 2\alpha_{max} \quad (9)$$

$$\alpha_{max} = 2 \times \arcsin \frac{\tau_{mez}}{\sigma_o} \quad (10)$$

Theoretically we can say (evidently if it is possible) that the most suitable adherent utilisation is in the cross section angle position in the range from α_{min} to α_{max}.

$$F_{mez\tau} = \tau_{mez} \times \frac{2S_o}{\sin 2\alpha} \quad (11)$$

From Eq. (6), we can determine the limiting tensile force for the individual cross section angles.

$$F_{mez\sigma} = \sigma_{mez} \times \frac{2S_o}{1 + \cos 2\alpha} \quad (12)$$

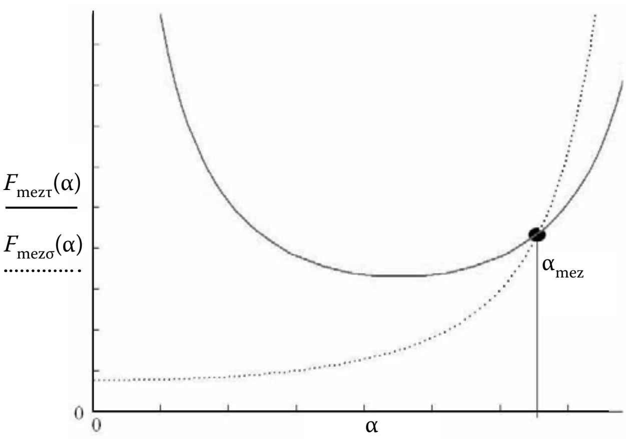

Figure 5. Graphical representation of the bonded joint limiting angle

If we express graphically (Figure 5) the relation between both limiting forces and the cross section angle, we find that up to the angle α_1 the joint must be dimensioned according to the limiting tensile force, over the angle α_1 according to the limiting shear force.

We determine the limiting angle α_1 from the limiting stresses equality.

$$\frac{\sigma_{mez}}{\tau_{mez}} = 1 \quad (13)$$

If we introduce Eqs. (5), (6) into the foregoing one, we get after modification the stress ratio.

$$\frac{\sigma_{mez}}{\tau_{mez}} = \frac{\frac{F}{S_o}\cos^2\alpha_I}{\frac{F}{S_o} \times \sin\alpha_I \times \cos\alpha_I} = \frac{\cos\alpha_I}{\sin\alpha_I} = \frac{1}{\mathrm{tg}\alpha_I} \quad (14)$$

From Eq. (14), we express the limiting angle sought.

$$\alpha_I = \mathrm{arctg} \frac{\tau_{mez}}{\sigma_{mez}} \quad (15)$$

RESULTS AND DISCUSION

Real load capacity determination of the wooden bonded scarf joint

Laboratory tests were carried out according to the standard CSN 66 8508 (1995) (Method of test for strength properties of adhesives in shear by tension loading (wood to wood)) using the recommended size of test specimens. The cross section surface was $S_0 = 300\ mm^2$. The shape and dimensions of the specimen are evident from Figures 6 and 7.

For the bonding of specimens from the spruce, the disperse solvent-type adhesive Herkules was used.

Tests were carried out for the determination of the tensile strength and shear strength, (Figure 4). The scarf influence on the bonded joint strength was also tested, (Figure 8). The bonded surfaces were prepared according to CSN 66 8508 (1995).

Characteristics of the chosen adhesive

Herkules pertains to significant disperse adhesives. In the Czech Republic it is commonly available. Polyvinyl acetate (PVAC) is the film-forming component. Polyvinyl acetate adhesives contain 50%–60% of solids. At a long-term storage or at bonded joints ageing a small amount of acetic acid is separated. This conduces to the chalk addition. Chalk removes

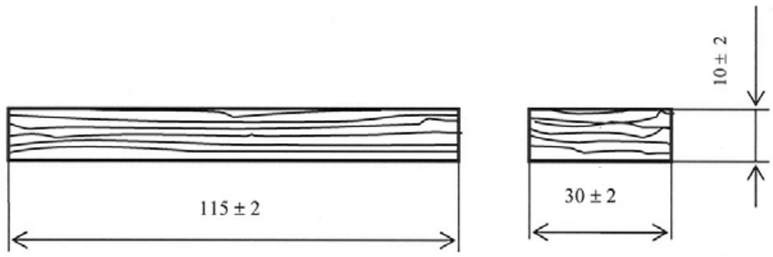

Figure 6. Shape and dimensions of the specimen for the tensile test

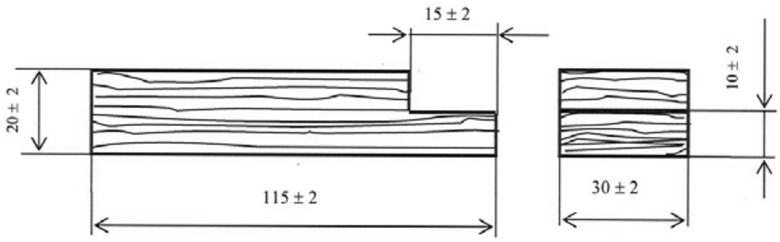

Figure 7. Shape and dimensions of the specimen for the shear test

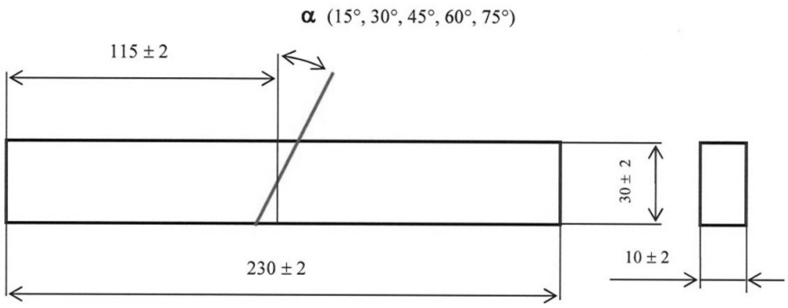

Figure 8. Shape and dimensions of the scarf spruce specimen

the acid trace amounts and above all it concentraces the mixture.

Methodical procedure of laboratory tests

Bonded spruce specimens were tested using the disperse solvent-type adhesive Herkules. The tensile tests and the shear tests were carried out. Then the relation between the scarf (15°, 30°, 45°, 60°, and 75°), (Figure 8), and the load capacity were tested. The bonded surfaces were prepared according to the recommendations presented in special literature.

The bonded specimens were homogenous, without defects and knots. The surface for the adhesive application in sufficient amount was clean, dry, and even. The adhesive was evenly spread. Then the second bonded piece was joined to the first one and fixed.

The specimens were cured for 48 hours at laboratory temperature. After curing, the bonded specimens were clamped symmetrically in the jaws of the universal tensile-strength testing machine ZDM 5 and the breakdown test was carried out. The feed rate of the jaw was 6 mm/min. After the failure, the maximal force was read and recorded. In the end, the data were evaluated.

Tests using 30 specimens were carried out for each of 6 scarf angles, thus total of 180 specimens were used. The calculated average force needed for the bonded joint failure at each scarf is presented in Table 1. The following limiting allowable stresses were determined: Pure tensile stress σ_{mez} = 3.85 MPa, pure shear stress τ_{mez} = 10.4 MPa. Table 1 presents the shear and normal stresses at the joint failure, calculated using Eqs. (5) and (6).

According to the foregoing table, the relation between shear/normal stresses and scarf angle is graphically represented in Figure 9. By interlay of individual points we get the Eqs.

$$\tau(\alpha) = 10^{-6}\,\alpha^4 - 4 \times 10^{-5}\alpha^3 - 0.0007\alpha^2 + 0.1061\alpha - 0.025 \quad (16)$$

$R^2 = 0.9997$

$$\sigma(\alpha) = 0.0006\alpha^2 + 0.0156\alpha + 3.9565 \quad (17)$$

$R^2 = 0.8914$

Using the measured values, we can get the relation between the failure force and the scarf angle

Table 1. Individual stress values at different scarf angles

α (°)	F (N)	τ (MPa)	σ (MPa)
0.00	1348.57	0.00	3.85
15.00	1617.14	1.19	4.31
30.00	1834.29	2.62	3.93
45.00	2157.14	4.35	3.08
60.00	4542.86	11.23	3.24
75.00	9614.29	26.51	1.84

α – scarf angle; F –loading force; τ– shear stress; σ – tensile stress

Table 2. Values of coefficient k in the dependence on the scarf angle

α (°)	k (–)
0.00	1.00
15.00	1.20
30.00	1.36
45.00	1.60
60.00	3.37
75.00	7.13

α – scarf angle; k – coefficient of the influence of the scarf angle

(Figure 10) at a bonded rectangular surface S_0 of 300 mm^2.

$$F(\alpha) = 0.0537\alpha^3 - 3.4418\alpha^2 + 66.8\alpha + 1319.5 \quad (18)$$

$R^2 = 0.9991$

For the practice, it is better to determine the coefficient k which indicates the influence of the scarf angle on the tensile stress. Then we can define the following relation:

$$\sigma_{red} = \sigma_d \times k \quad (19)$$

where:
σ_{red} – reduced tensile stress
σ_d – allowed normal stress for the given bonded joint

The coefficient k can be defined as the quotient of the failure force at the given scarf angle divided by the failure force at the scarf angle equal to zero.

$$k = \frac{F_\alpha}{F_0} \quad (20)$$

The values of the coefficient k are presented in Table 2.

From the measured values, we can also determine the Eq. of the relation between the coefficient k value and the scarf angle (Figure 11).

$$k(\alpha) = \left(\frac{\alpha^6}{2.5 \times 10^{10}} + 1\right) \times (-2.489 \times 10^{-4}\alpha^2 + 0.017\alpha + 1) \quad (21)$$

$R^2 = 0.9824$

CONCLUSIONS

Based on the tests described, the characteristics were determined of the bonded scarf joint load capacity. For the design of the bonded joint load capacity, the above mentioned characteristics can be used (Figures 9, 10, 11).

With a detailed view on the relation between the shear and normal stresses and the scarf angle, we can say that up to the value of the so-called limiting angle (15) is it necessary to determine the load capacity according to the normal stress allowed, and over the limiting angle according to the allowed shear stress. Our tests confirmed the validity of the theoretical relations of the limiting angle derived

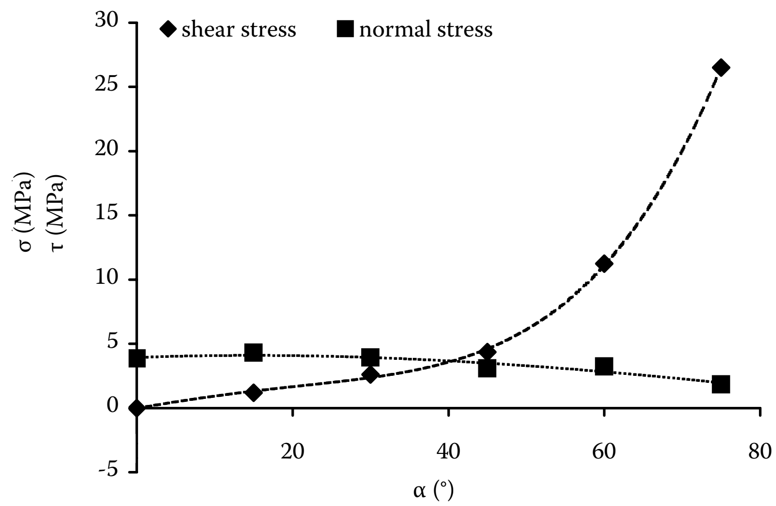

Figure 9. Relations between shear/normal stresses and scarf angle

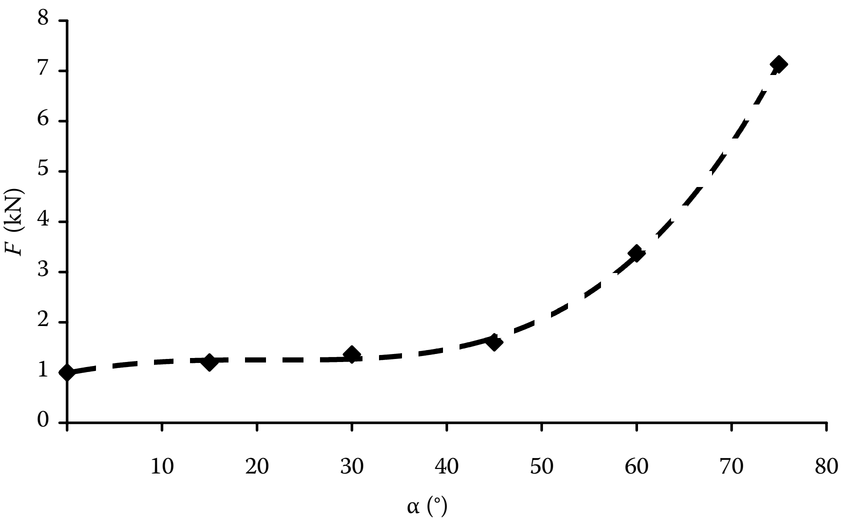

Figure 10. Relation between failure force and scarf angle at the bonded rectangular surface S_0 = 300 mm^2

from the Mohr's circle of stress. If we determine the limiting angle using these tests (Figures 5, 8) (it means the point where the curves of the normal stress and of the shear stress intersect), we find out the limiting angle value α_1 = 41°. But if we calculate the limiting angle value using Eq. (15) derived from Mohr's theory, we find out that the theoretical angle is

$$\alpha = \text{arctg}\,\frac{\tau_{mez}}{\sigma_{mez}} = \text{arctg}\,\frac{10.4}{3.85} = 69.9°$$

The difference between the theoretical and experimental values verified tests consists in several basic factors. In fact, the point is not the linear state of stress but the triaxial state of stress. The adhesive line behaves as a block of plastic material with non-covalent bonding. At loading, the deformation of the molecular chains occurs, and at the correctly chosen adhesive line thickness the adhesive line strength increases with regard to this deformation. Of course, the plastic deformation decreases. The classical Mohr's relations are not valid either. These relations were derived for the materials with covalent bonding. The stress distribution is not constant but is influenced by the intermolecular bonds between the bonded material and the adhesive line.

For practical calculation of the spruce bonded joints loading capacity, we can read the experimentally determined values of the allowable stress in the dependence on the scarf angle (Figure 9). For a simple calculation, the method using the coefficients of the scarf angle value can be used. These data are presented in the diagram (Figure 11).

When bonding wooden semi-products, it is necessary to accept the mechanical properties of wood which are very different in various directions. One of the most important properties of wood is its anisotropy (the properties of wood are different in various directions). Wood is a fibrous material and therefore it is much stronger in the longitudinal axis

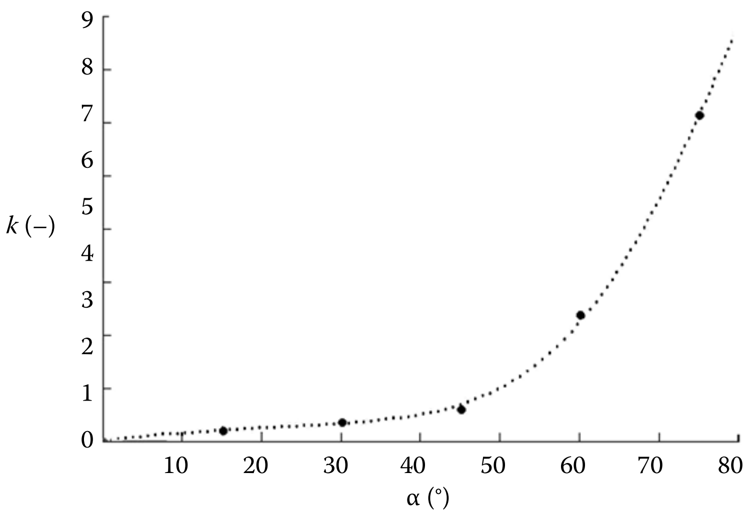

Figure 11. Values of the coefficient k in the dependence on the scarf angle

(in the fibres direction) than in the perpendicular (transversal) direction. Commonly, the strength of wood is sufficient especially at the tension in the fibres direction. The very strength of wood is influenced by a row of further significant factors (density, matter content, knots, wood defects etc.).

The tests carried out confirmed a great influence of the bonded surface size and of the stress type. It is useful to provide the butt joints with a scarf. In this way, the bonded surface increases. By the laboratory tests evaluation it was found that the scarfs of 60° and 75° are most useful for the bonded joint strength increase.

Using the disperse adhesive Herkules, the laboratory tests of spruce wood bonding were carried out. The presumption of the suitability of using the scarf joints was confirmed. By the tests, the scarfs of angles 60° and 75° were found as suitable for a substantial bonded joint strength increase.

References

ASTM (1994): ASTM D905-94 Standard Test Method for Strength Properties of Adhesive Bonds in Shear by Compression Loading. ASTM International, West Conshohocken.

CSN 66 8508 (1995): Test Methods for Wood Adhesives for Non-structural Parts. Determination of the Strength of Glued Connections in a Tensile Stress. UNMZ, Praha. (in Czech)

Franklin International (1998): Gluing and Furniture Design. Wood Adhesives Division, Industrial Product Group, Columbus.

Herák D., Chotěborský R., Müller M., Hrabě P. (2005): Bonded wooden balks. Research in Agricultural Engineering, **51**: 145–151.

Lorenz K. (2002): Supporting Structure II. ČVUT, Praha. (in Czech)

Morlier P. (1994): Creep in Timber Structures. [RILEM Report.] E & FN Spon, London.

Müller M. (2006): Adhesive bonding of metallic and non-metallic materials, [Ph.D. Thesis.] CULS, Prague. (in Czech)

Mukam F.J.A., Mouongue A. (2003): Strength of some wood adhesives used in Cameroo. International Journal of Adhesion and Adhesives, **23**: 287–291.

Němec J., Janáček V., Hurda B. (2005): Wood – Historical Lexicon. Grada Publishing, Praha. (in Czech)

Pizzi A. (1994): Advanced Wood Adhesives Technology. Dekker Inc., New York.

Pizzi A., Mitta K.L. (2003): Handbook of Adhesive Technology. Dekker Inc., New York.

Ševčík L. (2005): New methods of optimization machine details. Vědecká pojednání, Wissenenshaftliche Abhandlungen, Práce naukowe, **11**: 357–360.

Received for publication August 31, 2008
Accepted after corrections February 25, 2009

Abstrakt

Herák D., Müller M., Chotěborský R., Dajbych O. (2009): **Stanovení únosnosti dřevěných lepených spojů s úkosem**. Res. Agr. Eng., **55**: 76–83.

V článku je popsáno kompletní odvození teoretické únosnosti lepeného spoje s úkosem a sestrojení Mohrovy kružnice napjatosti pro jednoosou napjatost. Dále je v článku detailně vysvětlena metodika experimentálního stanovení únosnosti lepeného spoje. Součástí této práce je také stanovení skutečné únosnosti lepeného spoje s úkosem.

Klíčová slova: Mohrovy kružnice; jednoosá napjatost; lepení; úkosový spoj; dřevěný spoj

Corresponding author:

Ing. David Herák, Ph.D., Česká zemědělská univerzita v Praze, Technická fakulta, katedra mechaniky a strojírenství, Kamýcká 129, 165 21 Praha, Česká republika
tel.: + 420 224 383 186, fax: + 420 220 921 361, e-mail: herak@tf.czu.cz