Determination of loading capacity depending on bevel angle of the wooden bonded scarf joint

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Abstract

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The paper is focused on comparison of experimental and simple theoretical method of determination of loading capacity depending on bevel angle of wooden bonded scarf joint. The Mohr's circle principle, thus shear stress dependence on normal stress, is used for loading capacity formula derivation. It has been established that for random bevel angle under approximately 70 degrees the future loading capacity can be calculated from knowledge of ultimate force for bevel angle 0 and 90 degrees.

Keywords: scarf joint; bonded joint; wood; loading capacity; Mohr's circle

Wooden bonded joint design is based on experience followed by prototype testing, which increases costs and time cost of product development. On the other hand the methods based on intermolecular bond characteristics give very low accurate results (usually 50% and less) and they are absolutely unusable for practical exploitation (Peterka 1980; Qiao, Easteal 2001). Therefore a method, which predicts future loading capacity of designed joint simply enough and with sufficient accuracy, would be a good instrument for engineers.

The method derived in Herák et al. (2009) for loading capacity of wooden bonded scarf joint requires experimental determination of whole stress dependency on bevel angle of given wood material – adhesive combination to be made. Further simplification and generalization of the method was the goal of the paper.

MATERIALS AND METHODS

Wood is organic material composed of cells. It contains cellulose (circa 42%), hemicellulose (circa 26%), and lignin (circa 25%). The rest is composed of starch and fat (circa 1.8%), resins (circa 1.6%), proteins (circa 1%), and other minor components (Gibson, Ashby 1997). Typical wood materials, which are used in constructions and for instance in furniture industry, were taken as basic material for experiments; namely, *Picea abies*, *Pinus sylvestris*, *Larix decidua*, *Quercus robur*, *Tilia cordata* woods. Wood moisture content [determined by the Czech standard ČSN EN 13183-2 (2002)] was 12–16% depending on wood material type and experiment period, in view of the fact that experiments proceeded within several months.

Basic elements for samples were wooden blocks with following dimensions: 15 × 20 ± 1 mm, thus 300 mm^2

Table 1. Ultimate force (N)

	0°	15°	30°	45°	60°	75°	90°
Picea abies	1,658	1,689	2,056	2,861	4,056	5,879	2,056
Pinus sylvestris	1,935	2,132	2,659	3,867	5,286	5,698	2,549
Larix decidua	2,161	2,348	2,890	4,328	5590	5,982	2,164
Quercus robur	1,532	1,787	2,253	3,183	6,382	–	2,654
Tilia cordata	2,056	1,983	2,435	3,572	5,500	6,534	2,367

cross section, with the length of 200 ± 10 mm. These blocks were cut with saw under bevel angle of 0, 15, 30, 45, 60, 75, and 90 degrees. Then the surfaces of cuts were modified on angle cutter to get exact angles, purified and prepared by adhesive producer instructions and according to the Czech standard ČSN EN 205 (2003). Preparation of surfaces and layers of material adjacent to joint is fundamental for bond quality (Oberk et al. 2000). Twenty-one samples were prepared for each experimental period, thus 3 for each angle. Special equipment for set of 21 samples was designed to keep necessary time and force for proper hardening of joints. The bonding process took place in laboratory temperature 20–24°C according to given seasonal conditions, which was accordant to practical use of given adhesive (Motohashi et al. 1984).

Soudal 62A glue was used, which is adhesive based on PVAc (polyvinylacetate) dispersion. This type of adhesive is usually used in furniture industry and for joints with average load in unit production. Basic information about given adhesive can be found in its technical sheet. Main parameters are: specific mass approx. 1.1 g/cm^3, 45–47% of solids, minimal bonding temperature 4°C, bonding pressure 100–200 kPa, time of pressure application 2–4 h.

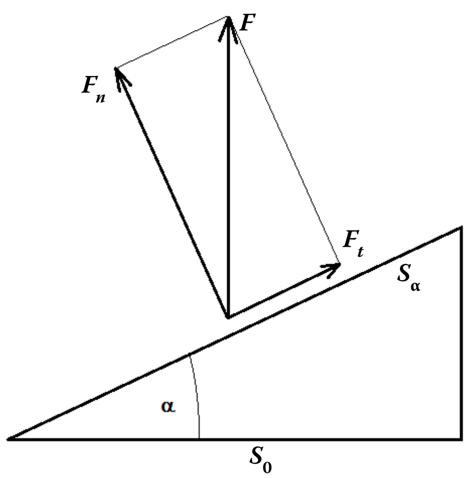


Fig. 1. Joint geometry and force distribution

Samples were brought to failure on shredder UTS 50 Testsysteme after bonding process completion. Speed of shredding process was 0.05 mm/s. Some samples, especially with smaller bevel angles, could be used again after affected material layer removal and surface re-preparation. Others had to be replaced with new ones due to vaster damage. The goal was to gain 10 valid values of force necessary for sample failure for each bevel angle. The experiment was considered as invalid when the value significantly missed the set of other values or the breach took place out of joint or in its small part. It is evident, that several hundred experiments had to be made.

RESULTS

For each bevel angle the arithmetic mean of ultimate force valid values was calculated. Results were determined in Table 1.

It is possible to notice missing value for *Quercus* wood for bevel angle 75 degrees in Table 1. Disadvantage of given method has shown in this case. Samples are attached in self-tightening clamps and with axial force increasing (with bevel angle – – bonded surface increasing) also the gripping pressure increases. *Quercus* wood is hard and its fibers are less flexible so they were crushed in clamps and the sample was mainly disrupted next to mounting point. It was not possible to gain 10 valid values for *Quercus* wood bonded at 75 degrees. Some fibers of other types of wood were also broken, but if the bonded joint fracture was not affected the experiment was considered as valid.

It is possible to derive following set of Eqs 1–5 from Fig. 1.

$$F_n = F \times \cos\alpha \quad (1)$$

$$F_t = F \times \sin\alpha \quad (2)$$

Table 2. Normal stress in joint surface accordant with sample failure (MPa)

	0°	15°	30°	45°	60°	75°	90°
Picea abies	5.53	5.25	5.14	4.77	3.38	1.31	0
Pinus sylvestris	6.45	6.63	6.65	6.45	4.41	1.27	0
Larix decidua	7.20	7.30	7.23	7.21	4.66	1.34	0
Quercus robur	5.11	5.56	5.63	5.31	5.32	–	0
Tilia cordata	6.85	6.17	6.09	5.95	4.58	1.46	0

where:
F – force necessary for sample failure (N)
F_n – normal component of force F (N)
F_t – tangential component of force F (N)
α – bevel angle (°)

$$S_\alpha = \frac{S_0}{\cos\alpha} \tag{3}$$

where:
S_α – (mm^2) surface of joint surface inclined under angle α (°)
S_0 – basic cross section of sample (mm^2)

$$\sigma_\alpha = \frac{F_n}{S_\alpha} = \frac{F}{S_0} \times \cos^2\alpha \tag{4}$$

$$\tau_\alpha = \frac{F_t}{S_\alpha} = \frac{F}{S_0} \times \sin\alpha \times \cos\alpha \tag{5}$$

where:
σ_α – normal stress (MPa) in scarf joint
τ_α – shear stress (MPa) in given joint
other parameters are accordant with previous equations

Normal and shear stress values acquired from Eqs 4 and 5 for each wood material and bevel angle were summarized in Tables 2 and 3 and then displayed graphically on Fig 2a,b.

The aim of the work was to gain a simple method to predict future loading capacity of designed joint in dependence on bevel angle with enough accuracy. Mohr's circle principle was used for this purpose. Thus graphical representation show dependency between normal and shear stress, whereas values for each type of stress can be read for any plane inclined under given angle as can be seen on Fig. 3.

Three functions were tested for simplifying given dependency by fitting experimentally gained points (Fig. 4). Program MathCAD 14 and its function genfit was used to obtain parameters. For results correlation analysis the function corr in the mentioned program was used. Following procedure is shown on *Picea* wood example.

Circle function with parameters generated by genfit function

General equation in the Cartesian coordinates for circle with center placed in coordinate origin Eq. 6 can be formulated as

$$r^2 = x^2 + y^2 \tag{6}$$

where:
r – radius
x, y – function variables

This can be evaluated for stress problem as

Table 3. Shear stress in joint surface accordant with sample failure (MPa)

	0°	15°	30°	45°	60°	75°	90°
Picea abies	0	1.41	2.97	4.77	5.85	4.90	6.85
Pinus sylvestris	0	1.78	3.84	6.45	7.63	4.75	8.50
Larix decidua	0	1.96	4.17	7.21	8.07	4.99	7.21
Quercus robur	0	1.49	3.25	5.31	9.21	–	8.85
Tilia cordata	0	1.65	3.51	5.95	7.94	5.45	7.89

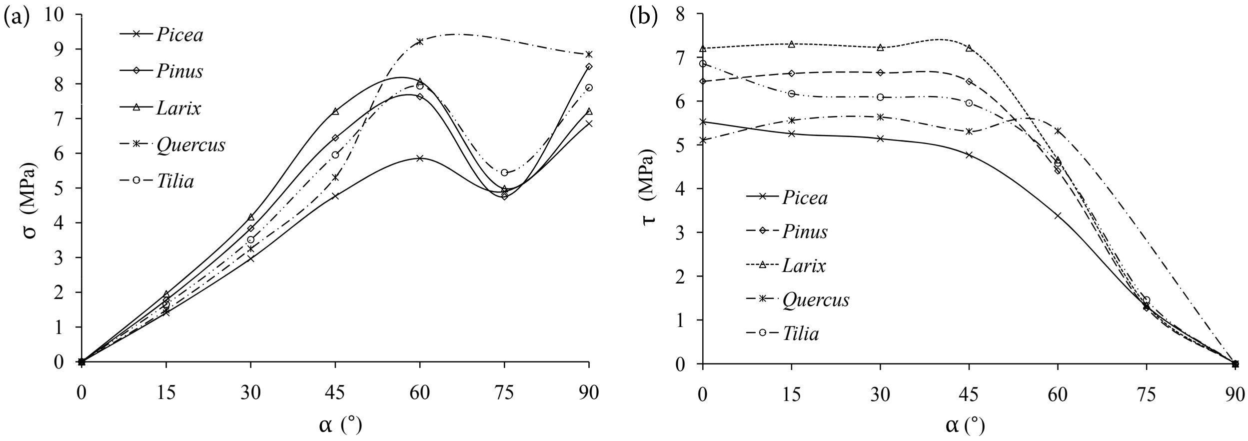


Fig. 2. Normal stress (a), shear stress (b) in joint surface accordant with sample failure

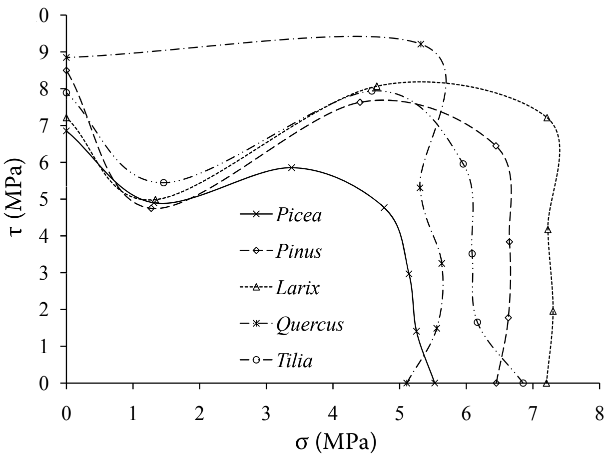


Fig. 3. Dependency between normal and shear stress

$$\tau = \sqrt{r^2 - \sigma^2} \qquad (7)$$

where:

τ – shear stress (MPa)

σ – normal stress (MPa)

r – radius (MPa) of circular representation of normal and shear stress dependency

Function genfit requires four parameters: set of x values, set of y values, result guess and vector containing function fitting on x and y values dependency and its partial derivative(s) by demanded function parameter(s).

Function corr returns Pearson's correlation coefficient, which squared represents coefficient of determination. This functions requires two parameters. First is the set of function values accordant with x values, second parameter is the set of real y values. Set of values is in MathCAD represented by one column matrix, where each element of set is represented by one row.

By using the mentioned program and functions following values were gained for radius r = 6.295 MPa with coefficient of determination R^2 = 0.78.

Ellipse function with parameters generated by genfit function (ellipse A)

General equation in the Cartesian coordinates for ellipse with center placed in coordinate origin can be formulated as

$$1 = \frac{x^2}{a^2} + \frac{y^2}{b^2} \qquad (8)$$

where:

a – semimajor axis of given ellipse

b – semiminor axis

x, y – function variables

This can be evaluated for stress problem as

$$\tau = \sqrt{b^2 \times (1 - \frac{\sigma^2}{a^2})} \qquad (9)$$

where:

τ – shear stress (MPa)

σ – normal stress (MPa)

a, b – semi axis (MPa) of elliptical representation of normal and shear stress dependency

In this case the program returned values a = 5.564 MPa and b = 7.067 MPa with coefficient of determination R^2 = 0.837.

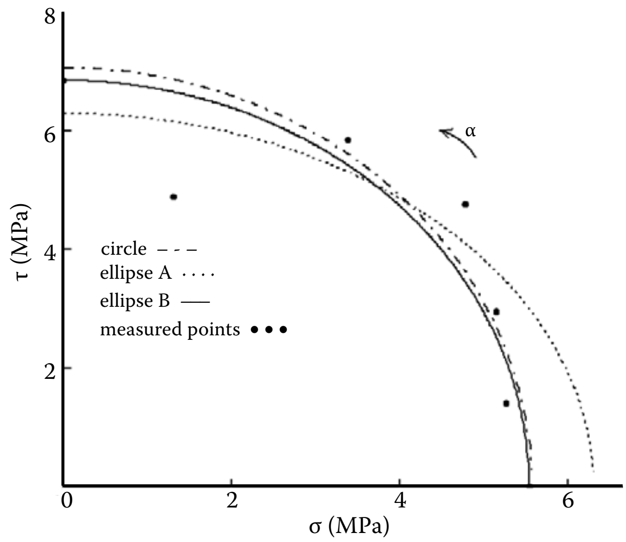


Fig. 4. Measured points and simplifying function variants

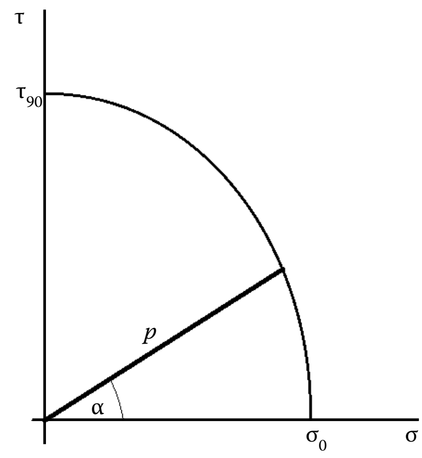


Fig. 5. Equation determination method

Ellipse function with semi axis equal to major stresses (ellipse B)

As semimajor axis we use directly value of normal stress for bevel angle of 0 degrees $a = \sigma_0 = 5.53$ MPa and as semiminor axis $b = \tau_{90} = 6.85$ MPa what is shear stress for bevel angle of 90 degrees, when MathCAD returned coefficient of determination $R^2 = 0.856$ for this function.

Obviously the elliptical function B is the simplest one to be gained from experimental results and enough accurate.

From general equations in parametric form for ellipse in canonical position Eqs 10 and 11, equations for stresses Eqs 12 and 13 can be determined.

$$x(\alpha) = a \times \cos\alpha \quad (10)$$

$$y(\alpha) = b \times \sin\alpha \quad (11)$$

where:
a, b – semi axis of ellipse
α – vector angle in polar coordinates
x, y – function values

$$\sigma(\alpha) = \sigma_0 \times \cos\alpha = \frac{F_0}{S_0} \times \cos\alpha \quad (12)$$

$$\tau(\alpha) = \tau_{90} \times \sin\alpha = \frac{F_{90}}{S_0} \times \sin\alpha \quad (13)$$

where:
$\sigma(\alpha)$ – normal stress (MPa) accordant with bevel angle α
$\tau(\alpha)$ – shear stress (MPa) accordant with bevel angle α
F_0 – measured force for bevel angle of 0 degrees which corresponds with normal stress σ_0 (MPa)
F_{90} – measured force for bevel angle of 90 degrees which corresponds with shear stress τ_{90} (MPa)
S_0 – basic cross section of sample (mm^2)
α – bevel angle (°)

Necessary condition for this conversion is that joint surface has to be equal for 0 and 90 degrees experiments. Let us have stress p parallel to axis of loading applied on joint surface as displayed on Fig. 5. Then Eq. 14 can be written.

$$p = \frac{\frac{F}{S_0}}{\cos\alpha} \quad (14)$$

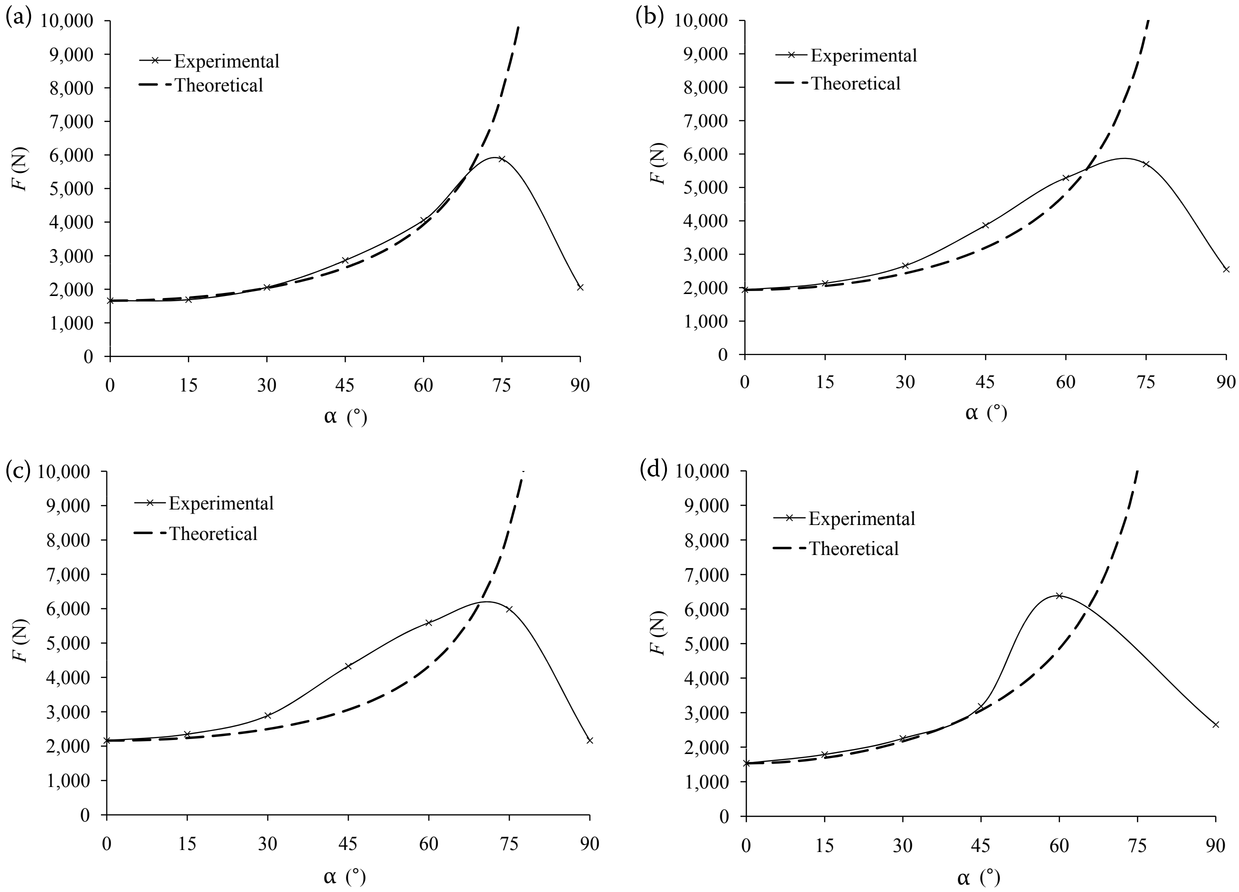
where:
p – previously mentioned axial stress (MPa)
S_0 – basic cross section of sample (mm^2)
α – bevel angle (°)

Stress p can be also read from given ellipse and can be calculated by Pythagorean theorem as shown in Eq. 15.

$$p = \sqrt{\sigma_\alpha^{\,2} + \tau_\alpha^{\,2}} \quad (15)$$

where:
p – axial stress (MPa)
σ_α – normal stress (MPa)
τ_α – shear stress (MPa) accordant with random bevel angle α

Thus after combining Eqs 12–15 we can write formula for probable future loading capacity of designed joint:



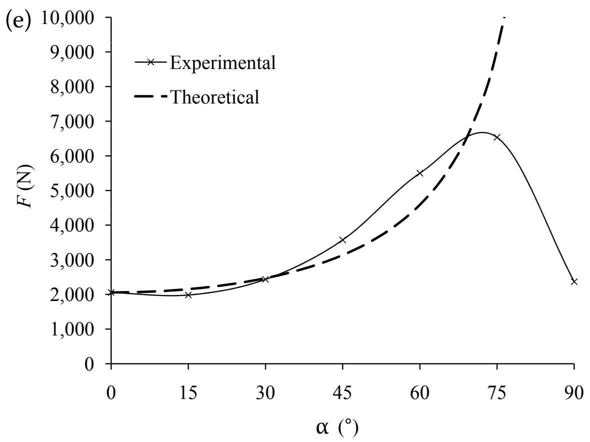


Fig. 6. Theoretical and experimental results comparison for *Picea* (a), *Pinus* (b), *Larix* (c), *Quercus* (d), *Tilia* (e) wood

$$F = \frac{\sqrt{{F_0}^2 \times \cos^2 \alpha + {F_{90}}^2 \times \sin^2 \alpha}}{\cos \alpha} \quad (16)$$

where:

F – theoretical force necessary to bring the sample with random bevel angle to failure

F_0 – force necessary to bring the sample with bevel angle of 0 degrees to failure

F_{90} – force to bring the sample with bevel angle of 90 degrees to failure and α is bevel angle

DISCUSSION

The experiment confirmed that adhesive joints design with shear loading dominance are preferable unlike tensile loaded ones (Özçifçi 2007; Malyshev, Salganik 1984).

From Fig. 2 (especially from Fig. 2b – tangential component of stress) a significant decrease of stress value in comparison with assumptive progression for angle of 75 degrees can be noticed. The most probable reason is that high angle of bevel the tips of bonded material are very sharp and thin. And also with regard to heterogeneity and anisotropy of wood load capacity of basic material in direction perpendicular to fibers is multiple times lower in contrast to parallel direction. This causes separation of thin tips of bonded parts, thus real active joint surface is smaller than theoretical.

When experimental results for bevel angle of 60 and 75 degrees are compared, bevel surface is approximately double (double adhesive consumption, space demand for joint), but real load capacity (force necessary for joint failure) increase is only approximately 50%. That implicates that designing joints with bevel angle of approx. 70 degrees and less are more advisable than higher values of the angle.

Fig. 6 shows theoretical and experimental results comparison. Theoretical force progression understandably converges to infinity with regard to cosine which converges to zero with angle approaching 90 degrees and is placed in denominator Eq. 6. On the other hand experiments for 90 degrees were made for finite joint surface concretely equal to cross section of used sample.

Theoretical function shows good accuracy for all wooden materials approximately up to bevel angle 70 degrees. In this section of dependency the experimental values are always approximately equal or higher than theoretical function. Thus the usage of Eq. 16 for future load capacity gives always "safe" results. The simplification consists only in experimental load capacity determination for two extreme cases and their substitution in Eq. 16.

CONCLUSION

It is obvious that derived method for determination of loading capacity depending on bevel angle of the wooden bonded scarf joint is usable for proper bevel angle. Due to wood material properties (heterogeneity, anisotropy) and different technological conditions and procedures, it is necessary to make final experimental verification on designed element prototype. However this method could be simplification and speedup in primary stadium of design process.

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