Tangent curve utilization for description of mechanical behaviour of pressed mixture

D. Herák[1], A. Kabutey[1], A. Sedláček[1], G. Gűrdil[2]

[1]*Department of Mechanical Engineering, Faculty of Engineering, Czech University of Life Sciences Prague, Prague, Czech Republic*
[2]*Department of Agricultural Machinery, Faculty of Agriculture, Ondokuz Mayis University Samsun, Samsun, Turkey*

Abstract

Herák D., Kabutey A., Sedláček A., Gűrdil G., 2011. **Tangent curve utilization for description of mechanical behaviour of pressed mixture**. Res. Agr. Eng., *57*: 13–18.

This article is focused on the utilization of tangent curve for description of mechanical behaviour of pressed mixture under compression loading. The aim of this experiment was to determine the general equation describing deformation characteristics of pressed mixtures under compression loading and to verify this equation. The experiment was carried out using mixture of seeds of common sunflower (*Heliantus annuus* L.), jatropha (*Jatropha curcas* L.), garden pea (*Pisum sativum* L.), common bean (*Phaseolus vulgaris* L.), and also mixture of spruce wood chips and waste paper chips. The results from the experiment showed that mechanical behaviour of the pressed mixtures under compression loading can be described by tangent curve function. From the determined coefficients of determination R^2 it was clear that fitted tangent curve functions described the measured amounts exactly for all pressed mixtures. It can be therefore assumed that this tangent curve function is appropriate for use in pressing any pressed mixtures during linear compression.

Keywords: compression; loading; jatropha; sunflower; bean; pea; wood; paper; chips; deformation; Levenberg; Marquardt

For more detailed understanding of the linear pressing process it is necessary to understand the mechanical behaviour of pressed mixture under compression loading (Fomin 1978; Blahovec, Řezníček 1980; Herák 2010). The pressed mixture during pressing is compressed in direction of the pressing and deformations in other directions are limited (Fig. 1). This type of pressing can be called linear pressing. The course of the deformation characteristics, dependency between compressive force and deformation, are in most cases determined by experiments and it can be described by general Eq. (1):

$$F(x) = f(x) \tag{1}$$

where:
F – compressive force
x – deformation of pressed mixture

Detailed knowledge describing the shape of the deformation characteristics is necessary to precisely determine the magnitude of the compressive force, strain energy, and modulus of elasticity in compression. The shape of these curves and its distribution in deformation diagram are the most important factors needed for accurate determination of pressing energy performance. Also, determined ranges of the pressing force magnitude depending on the deformation may help to design more appropriate and propose a complete technological process. Speci-

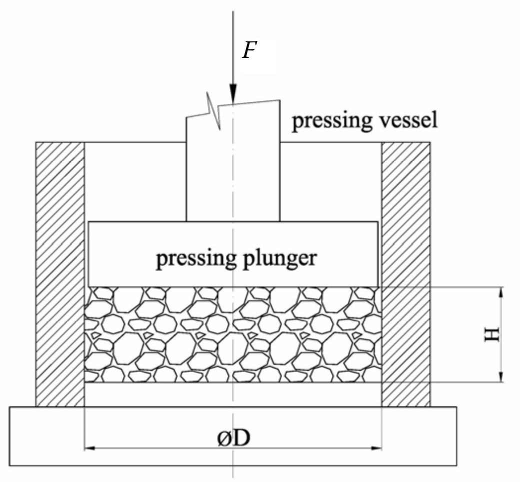

Fig. 1. Scheme of pressing equipment

fied dependency between volume energy and deformation can be used as one of the input amounts needed for complete energetic audit. For example, it was found that jatropha seeds (*Jatropha curcas* L.) after compression obtained deformation characteristics features similar to those of the strain tangential dependence (Herák 2010). Mechanical behaviour of the palm oil seeds (*Elaeis guineensis* L.) (Raji, Favier 2004) under compression loading is very similar to mechanical behaviour of jatropha seeds and similar to even distribution of strain fields (Owolarafe et al. 2007). Deformation characteristics of hazelnut (*Corylus avellana* L.), macadamia nut (*Macadamia integrifolia* L.) (Braga et al. 1999) or black pepper (*Piper nigrum* L.) (Murthy, Bhattacharya 1998) are also similar to jatropha deformation characteristic, only the middle part of the characteristics can be substituted by straight line dependence (Güner et al. 2003). Deformation characteristics of apricot seeds (*Prumus armeniaca* L.) and faba bean seeds (*Vicia faba* L.) can be generally described by the higher degree polynomial function (Lysiak, Laskowski 2004; Vursavus, Ozguven 2004). Deformation characteristics of coriander seeds (*Coriandrum sativum* L.), or seeds of African nutmeg (*Monodora myristica* L.) can be described by the curve with the S shape (Coskumer, Krabala 2006; Burubai et al. 2007). Seeds of safflower (*Carthamus tictorius* L.) (Baümler et al. 2006) have concave deformation characteristics in contrast to the convex characteristics of jatropha. The briquette from pit coal and oak and beech sawdust have mechanical characteristics similar to tangent curve dependency (Borowski 2009). Banana-peel briquettes produced with pressures ranging from 7–11 MPa have mechanical characteristics which can be described by third degree polynomial function (Wilaipon 2007, 2009). For briquette from energetic herbs it was found that the courses of mechanical characteristics have linear shape (Plíštil et al. 2005). The aim of this experiment was to determine the general equation describing deformation characteristics of pressed mixture of common sunflower, jatropha, garden pea, common bean, wood chips, and waste paper chips under compression loading and to verify this equation.

MATERIALS AND METHODS

Sample

For this experiment intact and undamaged seeds following plants were used: common bean (*Phaseolus vulgaris* L.), garden pea (*Pisum sativum* L.), common sunflower (*Heliantus annuus* L.), jatropha (*Jatropha curcas* L.). Spruce wood chips and waste paper chips were also included. Ten experimental samples for each compressed mixture were prepared from these seeds and chips and their physical properties are presented in the table (Table 1). The moisture content *Mc* (% d.b.) of the pressed mixtures was determined using the ASTA method (1985) (Olaniyan, Oje 2002), the weight of the mixtures m_s (g) was determined using equipment Kern 440–35N (Kern & Sohn GmbH, Stuttgart, Germany) and the porosity P_f (%) was calculated from bulk and true densities using the relationship given by porosity formula Eq. (2) (Sirisomboon et al. 2007):

$$P_f = \left(1 - \frac{\rho_b}{\rho_t}\right) \times 100 \qquad (2)$$

where:

P_f – porosity (%)

ρ_b – bulk density (kg/m^3)

ρ_t – true density (kg/m^3)

The bulk density, ρ_b was determined as the weight of the sample divided by volume of pressing vessel 3.629×10^{-4} m^3. The true density, ρ_t was determined for average moisture content of each sample from the previous experiments as jatropha 971 kg/m^3 (Sirisomboon et al. 2007), common sunflower 885 kg/m^3 (Isik, Izli 2007), garden pea 1,150 kg/m^3 (Yalçýn et al. 2007), common bean 1,370 kg/m^3 (Ozturk et al. 2009), waste paper chips 700 kg/m^3 (Vieira, Rocha 2007), and spruce wood chips 650 kg/m^3 (Lehtikangas 2001).

Table 1. Determined physical properties of selected pressed mixtures

Pressed mixture	m_s (g)	P_f (%)	Mc (% d.b.)
Common bean	264.6 ± 6.1	46.78 ± 1.22	9.15 ± 0.23
Jatropha	126.9 ± 3.1	63.99 ± 0.99	5.39 ± 0.65
Garden pea	274.3 ± 4.2	34.28 ± 0.98	10.3 ± 0.45
Common sunflower	149.8 ± 4.6	53.36 ± 1.43	8.45 ± 0.56
Wood chips	38.4 ± 3.5	83.72 ± 1.40	6.62 ± 0.69
Waste paper chips	76.2 ± 3.5	70.01 ± 1.38	3.22 ± 0.43

m_s – weight of the sample; P_f – porosity of the sample; Mc – moisture content of the sample

Compression test

To determine the development of the magnitude of the pressing force depending on the linear deformation, the ZDM 50 – 2313/56/18 (VEB, Dresden, Germany) pressing device was used to record the course of deformation function in an exact and analogical manner. Other devices including a pressing plunger and pressing vessel with inner diameter D = 76 mm (Fig. 1) were also used. The mixture of samples, that is individual samples plus air space, were placed into the pressing vessel and height of the mixture layers was measured to be H = 80 mm. The mixtures were pressed under the temperature of 20°C and the pressing rate was v = 1 mm/s. The experiment was repeated for each pressed mixture 10 times. The measuring range of force was between 0 N and 250 kN, in which the test was stopped. Individual points of measurement were digitally recorded and analyzed with each new addition of deformation of 0, 5 mm. The computer program MathCAD 14 and its function genfit, which uses Levenberg-Marquardt algorithm (Pritchard 1998), was used for data fitting.

Approximation of deformation curves

Tangent curve, which was originally used to describe the mechanical behaviour of the jatropha seeds under compression loading (Herák 2010), was modified as a general curve suitable to describe the deformation characteristics. This curve can be described by the following Eq. (3):

$$F(x) = A \times \left[tan(B \times x)\right]^n \quad (3)$$

where:
F – compressive force (N)
A – force coefficient of the mechanical behaviour (kN)
B – deformation coefficient of the mechanical behaviour (1/mm)
x – deformation (mm)
n – exponent of the function (–)

This function has a simple derivation and integration, which can be used to determine the strain energy and modulus of elasticity in compression. With aid of derivations of this function it can be used to find an appropriate method for regression using the Levenberg-Marquardt algorithm (Marquardt 1963), which provides numerical solutions to the problem of minimizing deviations in general nonlinear functions over space of function parameters. The Levenberg-Marquardt algorithm is an iterative technique that locates the minimum of a function that is expressed as the sum of squares of nonlinear functions. It has become a standard technique for nonlinear least-squares problems (Lourakis 2005). The Levenberg-Marquardt algorithm utilizes derivation of function by searched coefficients for determining these coefficients of the function for fitting measured data. These derivations can be generally described by Eq. (4) and Eq. (5):

$$\frac{dF(x)}{dA} = \left[tan(B \times x)\right]^n \quad (4)$$

$$\frac{dF(x)}{dB} = A \times n \times x \times \left[tan(B \times x)\right]^{n-1} \times \left[\left(tan(B \times x)\right)^2 + 1\right] \quad (5)$$

These Eq. (4) and Eq. (5) can be mathematically adjusted to the shape Eq. (6) and Eq. (7) which contain compressive force function Eq. (3):

$$\frac{dF(x)}{dA} = \frac{F(x)}{A} \quad (6)$$

$$\frac{dF(x)}{dB} = n \times x \times \left[\frac{F(x)^{\frac{n+1}{n}}}{A^{\frac{1}{n}}} + \frac{F(x)^{\frac{n-1}{n}}}{A^{\frac{-1}{n}}}\right] \quad (7)$$

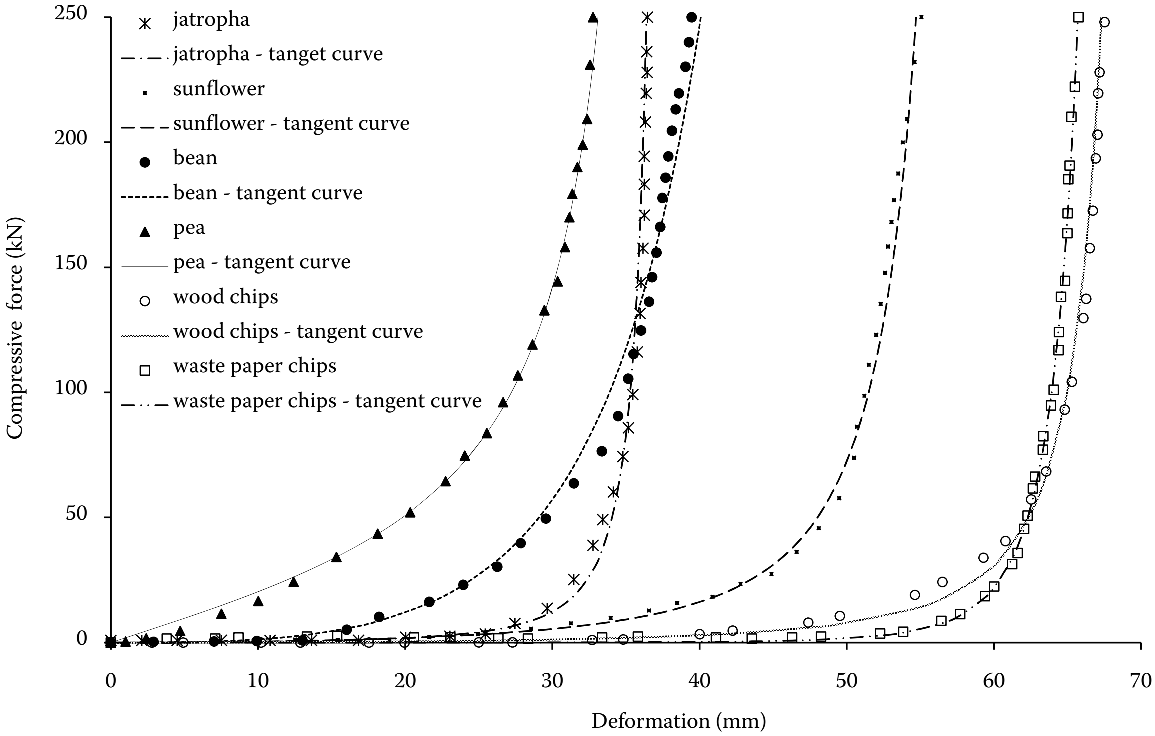

Fig. 2. Measured amounts of mechanical characteristics of selected pressed mixtures and their fitted functions

Such modified equations allow finding the coefficients without calculating trigonometric functions; this expression of the Eqs (4) and (5) is essentially a polynomial function. These modified equations for fitting of measured data can be optimal for Levenberg-Marquardt process which is utilized in mathematical software MathCAD 14. This software works with "genfit" function which operates with the Levenberg-Marquardt algorithm described above.

RESULTS AND DISCUSSION

The amounts determined from individual experiments are shown in Fig. 2. Measured amounts were fitted by tangent line function Eq. (3) whose coefficients of mechanical behaviour and determination are shown in Table 2. Graphical displaying of measured amounts and fitted functions for each pressed mixture are displayed in the graph (Fig. 2). From the determined coefficients of determination R^2 in Table 2 it was found that fitted functions described the measured amounts accurately. The amounts of the coefficients of determination in all experiments were close to one. From analysis of the images of individual experiments it was clear that the fitted curves described accurately measured amounts over the whole range of deformation. The validity of these equations is limited from zero to maximum deformation of pressed mixture. From the course of individual deformation characteristics it is evident that the force coefficient of the mechanical behaviour A (kN) influenced the slope of the deformation characteristics and deformation coefficients of the mechanical behaviour B (1/mm) also influenced the range of deformation, the product of these two coefficients A/B (kN/mm) is essentially the initial rigidity of the system, exponent n (–) determines the curvature of a tangential function. From the fitted functions graphically displayed in Fig. 2 it was clear that these functions precisely described the measured amounts in all regions of deformation characteristics. From the statistical analysis ANOVA which was calculated in the MathCAD 14 software for the level of significance 0.05, it was seen that the values of F_{crit} (critical value compares a pair of models) were higher than F_{ratio} values (value of the F-test) for all measured pressed mixtures and amounts of P value (significance level at which it can be rejected by the hypothesis of equality of models) were higher than significance level 0.05. This shows that tangent curves can be used for fitting measured amounts since relationships between measured amounts and tangent curve amounts were statistically significant. All values of F_{crit}, F_{ratio}, and P are presented in Table 2. Determined tangent curve function Eq. (3) conforms not only to their course but also to the boundary

Table 2. Determined coefficients for selected pressed mixtures

Pressed mixture	*A* (kN)	*B* (1/mm)	*n* (–)	F_{crit} (–)	F_{ratio} (–)	*P* (–)	R^2 (–)
Commnon bean	84.63	0.024	3	4.001	0.00476	0.945	0.991
Jatropha	1.55	0.041	2	4.007	0.196	0.659	0.991
Garden pea	45.57	0.042	1	4.034	0.00052	0.982	0.999
Common sunflower	5.58	0.026	2	3.991	0.067	0.796	0.994
Wood chips	2.10	0.021	4	3.996	0.0053	0.942	0.997
Waste paper chips	0.14	0.022	2	3.955	0.0018	0.968	0.997

A – force coefficient of the mechanical behaviour; *B* – deformation coefficient of the mechanical behaviour; *n* – exponent of the deformation curve; F_{crit} – critical value that compares a pair of models; F_{ratio} – value of the *F*-test; *P* – significance level at which it can be rejected the hypothesis of equality of models; R^2 – coefficient of determination

conditions and to physical essence of searched function coefficients The graphs show that the pressing process can be segmented into three regions (Herák 2010). Within the first region, the modulus of elasticity in compression constitutes a constant and the dependency between the stress and deformation is a linear one; in fact, this is an analogy to the Hooke law, but defined for a non-homogeneous and anisotropic material. The second phase presents a region in which the pressed mixtures behave as porous materials with molecular cohesion bond. During the third final phase, the deformed mixtures behave as anisotropic non-homogeneous materials as well. The modulus of elasticity can be considered a constant; again, similarities to the Hooke law occur in this region. From the calculated amounts of porosity presented in Table 1, it can be noticed that deformation characteristics can also be influenced by porosity due to the fact that higher value of porosity means higher compression of the pressed mixture and the pressing process needs a longer compression time. The most affected part of the pressing by porosity is the first region in deformation characteristics already described above. It can also be expected that the boundary between region I and II in deformation characteristics are generally influenced by amount of porosity. This assumption is evident from the calculated values of the porosity (Table 1) and from the mechanical characteristics (Fig. 2). The next property affecting mechanical behaviour of the pressed mixture under compression loading is the weight of the sample (Table 1). From determined amounts and deformation characteristics it was observed that greater weight of the sample means smaller compression of the pressed mixture. Comparing the porosity values calculated for the selected pressed mixtures with their maximum deformations, a positive linear correlation was found and thus an increase in porosity value showed an increase in maximum deformation. However, based on the model data there was no simple correlation between the force coefficients of the mechanical behaviour, deformation coefficients of the mechanical behaviour, exponents of the deformation curve of the selected pressed mixtures and porosity. It is possible that these coefficients could probably depend on porosity but for determining this correlation it would be necessary to set up a more dimensional model, which contains all coefficients, and this is suggested for further study.

CONCLUSION

It was found that the determined modified equation of tangent curve function can be used for describing the mechanical behaviour of the mixtures of plant seeds, oil bearing crops, wood chips, and waste paper chips under compression loading. Tangent curves can be used for fitting measured amounts since relationships between measured amounts and tangent curve amounts were statistically significant. Generally, this tangent curve function could be appropriate for use in pressing any pressed mixtures during linear compression. The study also revealed that distribution of pressing regions and general shape of deformation curve is influenced by the amount of porosity, by initial rigidity of the pressed mixture and indirectly by weight of the pressed sample. Again it was found that the greater the porosity values the longer the compression of the pressed mixture, and the greater the weight of the sample the smaller the compression of the pressed mixture. Porosity and maximum deformation of selected pressed mixtures showed linear positive correlation and thus an increase in porosity increased the deformation. However, based on the model data there was no correlation between the defined coefficients and porosity.

References

Baümler E., Cuniberti A., Nolasco S.N., Riccobene I.E., 2006. Moisture dependent physical and compression properties of safflower seed. Journal of Food Engineering, *72*: 134–140.

Blahovec J., Řezníček R., 1980. Frakcionace píce (Fractionation of the forage). Prague, Czech University of Life Sciences Prague: 370.

Borowski G., 2009. Possibilities of utilization of energy briquettes. In: 8th EEEIC Conference, May 10–13, 2009. Karpacz: 107–110.

Braga G.C., Couto S.M., Hara T., Neto J.T.P.A., 1999. Mechanical behavior of macadamia nut under compression loading. Journal of Agriculture Engineering Research, *72*: 239–245.

Burubai W., Akor A.J., Igoni A.H., Puyate Y.T., 2007. Fracture resistance of African nutmeg (*Monodora myristica*) to compressive loading. Electronic Journal of Environmental, Agricultural and Food Chemistry, *6*: 2434–2442.

Coskumer Y., Karababa E., 2006. Physical properties of coriander seeds. Journal of Food Engineering, *80*: 408–416.

Fomin V.I., 1978. Vlažnoje frakcionirovanije zeljonych kormov (Wet Fractionation of Green Forage). Rostov on Don, Russia, Izdatelstvo Rostovskovo Universiteta: 160.

Güner M., Dursun E., Dursun G., 2003. Mechanical behaviour of hazelnut under compression loading. Biosystems Engineering, *85*: 485–491.

Herák D., Gürdil G., Sedláček A., Dajbych O., Simanjuntak S., 2010. Energy demands for pressing *Jatropha curcas* L. seeds. Biosystems Engineering, 106: 527–534.

Isik E., Izli N., 2007. Physical properties of sunflower seeds (*Helianthus annuus* L.). International Journal of Agricultural Research, *2*: 677–686.

Lehtikangas P., 2001. Quality properties of pelletised sawdust, logging residues and bark. Biomass and Bioenergy, *20*: 351–360.

Lourakis M.I.A., 2005. A Brief Description of the Levenberg-Marquardt Algorithm Implemented by Levmar. Crete, Greece, ICS FORTH, Heraklion: 2

Lysiak G., Laskowski J., 2004. Investigation of mechanical properties of faba bean for grinding behavior prediction. Acta Agrophysica, *4*: 753–762.

Marquardt D.W., 1963. An algorithm for the least-squares estimation of nonlinear parameters. SIAM Journal on Applied Mathematics, *11*: 431–441.

Murthy C.T., Bhattacharya S., 1998. Moisture dependent physical and unaxial compression properties of black pepper. Journal of Food Engineering, *37*: 193–205.

Olaniyan A.M., Oje K., 2002. Some aspects of the mechanical properties of shea nut. Biosystems Engineering, *81*: 413–420.

Owolarafe O., Osunleke A.S., Oyebamij B.E., 2007. Effect of hydraulic press parameters on crude palm oil yield. International Agrophysics, *21*: 285–291.

Ozturk I., Kara M., Elkoca E., Ercisli S., 2009. Physico-chemical grain properties of new common bean cv. 'Elkoca-05'. Scientific Research and Essay, *4*: 88–93.

Plíštil D., Brožek M., Malaťák J., Roy A., Hutla P., 2005. Mechanical characteristics of standard fuel briquettes on biomass basis. Research in Agricultural Engineering, *51*: 66–72.

Pritchard P.J., 1998. MathCAD: A Tool for Engineering Problem Solving. New York, McGraw-Hill Science Engineering: 336.

Raji A.O., Favier J.F., 2004. Model for the deformation in agricultural and food particulate materials under bulk compressive loading using discrete element method II: Theory, compression of oilseeds. Journal of Food Engineering, *64*: 373–380.

Sirisomboon P., Kitchaiya P., Pholpo T., Mahuttanyavanitch W., 2007. Physical and mechanical properties of *Jatropha curcas* L. fruits, nuts and kernels. Biosystems Engineering, *97*: 201–207.

Vieira M.G.A., Rocha S.C.S., 2007. Drying conditions influence on physical properties of recycled paper. Chemical Engineering and Processing, 46: 955–963.

Vursavus K., Ozguven F., 2004. Mechanical behaviour of apricot pit under compression loading. Journal of Food Engineering, *65*: 255–261.

Wilaipon P., 2007. Physical characteristics of maize cob briquette under moderate die pressure. American Journal of Applied Sciences, *4*: 995–998.

Wilaipon P., 2009. The effects of briquetting pressure on banana-peel briquette and the banana waste in Northern Thailand. American Journal of Applied Science, 6: 167–171.

Yalçýn Ý., Özarslan C., Akbaţ T., 2007. Physical properties of pea (*Pisum sativum*) seed. Journal of Food Engineering, *79*: 731–735.

Received for publication April 13, 2010
Accepted after corrections July 4, 2010

Corresponding author:

doc. Ing. David Herák, Ph.D., Czech University of Life Sciences Prague, Faculty of Engineering, Department of Mechanical Engineering, Kamýcká 129, 165 21 Prague, Czech Republic
phone: + 420 224 383 186, fax: + 420 234 381 815 , e-mail: herak@tf.czu.cz