Comparison of numerical integration methods in strapdown inertial navigation algorithm

V. Cviklovič, D. Hrubý, M. Olejár, O. Lukáč

Faculty of Engineering, Slovak University of Agriculture in Nitra, Nitra, Slovak Republic

Abstract

Cviklovič V., Hrubý D., Olejár M., Lukáč O., 2011. **Comparison of numerical integration methods in strapdown inertial navigation algorithm**. Res. Agr. Eng., *57 (Special Issue)*: S30–S34.

The numerical mathematical theory provides a few ways of numerical integration with different errors. It is necessary to make use of the most exact method with respect to the computing power for a majority of microprocessors, because errors are integrated within them due to the algorithm. In our contribution, trapezoidal rule and Romberg's method of numerical integration are compared in the velocity calculation algorithm of the strapdown inertial navigation. The sample frequency of acceleration and angular velocity measurement was 816.6599 Hz. Inertial navigation velocity was compared with precise incremental encoder data. Trapezoidal method velocity error in this example was 1.23×10^{-3} m/s in the fifteenth-second measurement. Romberg's method velocity error was 0.16×10^{-3} m/s for the same input data.

Keywords: Romberg's method; trapezoidal rule; accelerometer; gyroscope; micro electro-mechanical systems (MEMS)

The utilisation of navigations has recently expanded to various scientific and engineering departments. We encounter the GPS system support most frequently, with it being increasingly available to general public. This system is used in such spaces, where the reception of the satellite signals is possible. However, there are some examples when we need to use navigation in closed spaces, for instance a storage house, where this signal is not available. In such cases, it is possible to use the advantages of inertial navigation, which does not require any input electric or magnetic signals, because the information about the position is obtained through the acceleration and gyro data from accelerometers and gyroscopes.

Two basic principles exist of inertial navigation. The first principle utilises gimbal suspension with a gyroscopically stabilised platform for balancing the sensors with predefined reference casing. Some advantages of the presented navigation include: a lower power strain of the sensor and a simpler calculation of the actual position. The navigation system without gimbal suspension is placed on a surface which is tightly connected to a vehicle (Titterton, Weston 2004).

We utilise the integrated circuit sensors in this principle, which causes inclination towards this technology. It concerns a combination of electronics and the 3D mechanical microelements, which convert the measured variable into an electrical signal. Consequently, this signal is quantified and sampled. The digital value is available on the output. The common title for this sensor subgroup is micro electro-mechanical systems (MEMS).

MATERIAL AND METHODS

The most important knowledge by inertial navigation is the acceleration value in a correct period (at the exactly defined moment). The acceleration is measured in each axis of the 3D Cartesian system. The accelerometer axes are consistent with the axes
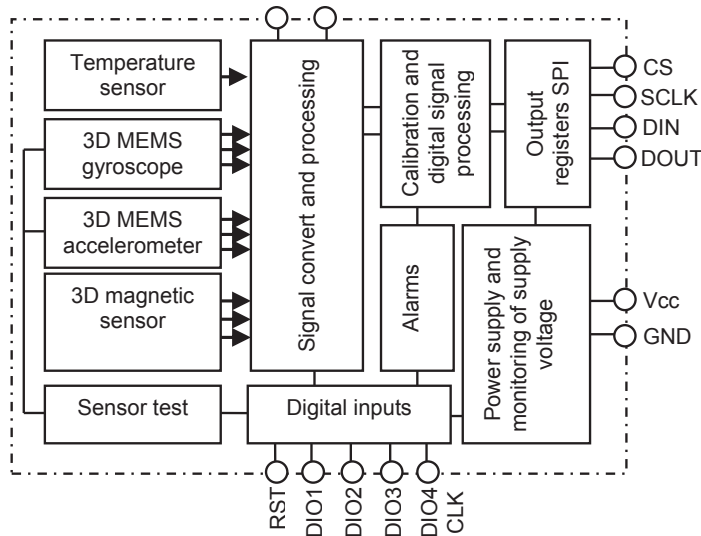
Fig. 1. Block diagram of the accelerometer ADIS16405 (Analog Devices 2009)
RST – device reset, DIO1-4 – digital general purpose pins, CLK – synchronisation pulse, GND – ground, Vcc – supply voltage, CS – chip select, SCLK – communication clock, DIN – data input, DOUT – data output

of the relative system by way of illustration. Thus we avoid complicated mathematics, which does not affect the position determination and the position error, if the information from the gyroscope and inclinometer are not taken into account.

In our example of inertial navigation, MEMS 3 axial inertial sensor ADIS16405BMLZ (Analog Devices, Inc., Norwood, USA) with digital output was used. Block diagram of the sensor is shown in Fig. 1. The capacitance sensors are sensitive parts of the accelerometers. In the case of the correctly calibrated sensor being in the rest position, the absolute value of the gravity acceleration is available at the output (Analog Devices 2009).
Acceleration is a second-order derivative of trajectory by time. The point position is calculated (can be expressed) by the position vector of the mass point:

$$\int_{r_0}^{\vec{r}} d\vec{r} = \int_0^t \vec{v}_0 \times dt + \int_0^t (\vec{a}t) \times dt = \vec{v}_0 \int_0^t dt + \vec{a} \int_0^t t \times dt \quad (1)$$

The position change is then the difference between the vectors:

$$\vec{r} - \vec{r}_0 = \vec{v}_0 t + \frac{\vec{a}t^2}{2} \quad (2)$$

where:
$\vec{r}$ – position vector (m)
$\vec{v}$ – velocity vector (m/s)
$\vec{a}$ – acceleration vector (m/s^2)
t – time (s)
$\vec{v}_0$ – initial velocity vector (m/s)

It follows that the acquired information on the position is set by double integral acceleration, consequently the global position error will integrate in

time. To minimise these errors, a properly configured Kalman's filter is used. All but one integrated methods bring errors into the calculations. The integrals for the actual position calculation cannot be calculated analytically, neither can they be defined by elementary functions. In some cases symbolic solutions exist, but this is more demanding than the numerical integration (Grewall et al. 2007).

By numerical integration, we may select from two basic alternatives. Either the interpolating polynomials are integrated or other integration methods are used. The interpolating upper (higher) degree polynomials are integrated with a greater error.

To calculate the definite integral of function *f(x)* given by the functional values in the equidistant points the Newton-Cotes quadrature formulas are most frequently used. The best known applicable formulas are the trapezoidal rule and Simpson's rule. The outstanding feature of the trapezoidal rule is a slow convergence of the numerical process at relatively low accuracy and high error h^2 (Mošová 2003):

$$R_L = -\frac{(b-a)^3}{12n^2} f''(\xi) \quad (3)$$

where:
$\xi \in \langle a, b \rangle$ – derivation of function in interval $\langle a, b \rangle$
R_L – method error
a, b – integral range
n – samples quantity in the interval $\langle a, b \rangle$

Simpson's rule approximates the selected function more precisely. The integral value of *f(x)* is calculated in the node points. This method is characterised by a small error h^4. Geometrical interpretation is the sum of the areas above triplet of node points
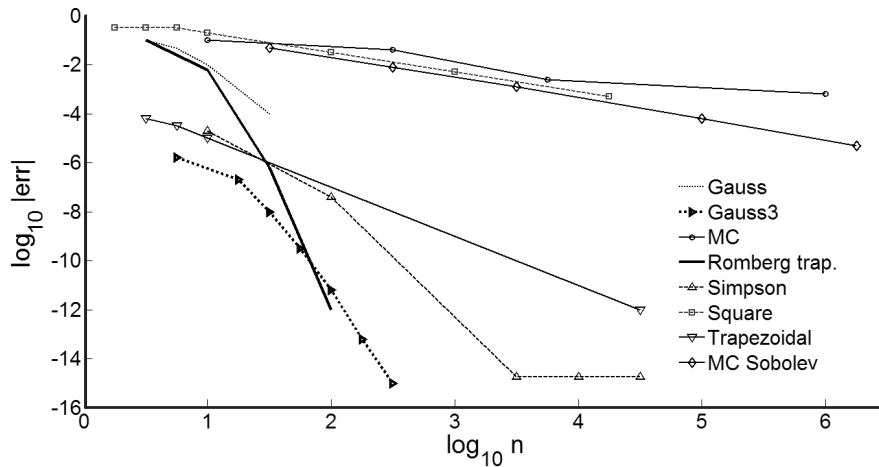
Fig. 2. Relative error comparison of the integration methods according to the amount of the points used (VICHER 2003)

with the common (collective) threshold point for the adjacent parabolas. The error of the calculation method is:

$$R_S = -\frac{(b-a)^5}{12n^4} f^{(4)}(\xi) \tag{4}$$

The precision of calculation depends on quantize step h. This is valid generally, not only for the trapezoidal and Simpson's rules. The calculations with the highest point count (amount) in the range $\langle a, b \rangle$ are the most exact. Since in this case is it needed to calculate the integrate area using algorithm in a microcontroller, the Romberg's method, that uses the calculation way of definite integral on the basis of trapezoidal rule, shows to be the most efficient and the most exact. Romberg showed by means of this method that it is possible to utilise the methods with greater errors for the formulation of more precise approximate integral value when using approximation for equal integration range with various partition steps. By combining the result with a higher error, the result with a lower error is reached (VICHER 2003).

With this method, we approximate first of all the integral in the interval $a \leq x \leq b$, that is split into $8N$ parts (N is integer, which is defined by the number of nodes) with step $h = (b - a)/8N$. With the aid of the node points $f(a + kh)$, where $k = 1, 2, ... , 8N{-}1$, non-precision approximation is obtained when using Eq. 5. According to Eqs 6 and 7, other approximations of the integral are calculated, by means of which the step value gradually decreases from $8h$ to h for value T_8.

$$T_1 = 8h \sum_{m=0}^{N-1} F_{8m} = (b-a) \overline{\sum_{m=0}^{N-1} F_{8m}} \tag{5}$$

$$U_1 = 8h \sum_{m=0}^{N-1} F_{8m+4} = (b-a) \overline{\sum_{m=0}^{N-1} F_{8m+4}} \tag{6}$$

$$U_2 = (b-a) \overline{\sum_{m=0}^{N-1} F_{8m+2} + F_{8m+6}} \tag{7}$$

$$U_4 = (b-a) \overline{\sum_{m=0}^{N-1} F_{8m+1} + F_{8m+3} + F_{8m+5} + F_{8m+7}} \tag{8}$$

$$T_2 = \overline{T_1 + U_1}, \; T_4 = \overline{T_2 + U_2}, \; T_8 = \overline{T_4 + U_4} \tag{9}$$

U and T are rough approximations of the calculated integral with the error rank h^2. Further, the more precise approximation using equations is sought (KAČEŇÁK 2001):

$$S_2 = T_2 + \frac{T_2 - T_1}{2^2 - 1} \tag{10}$$

$$S_4 = T_4 + \frac{T_4 - T_2}{2^2 - 1} \tag{11}$$

$$V_2 = U_2 + \frac{U_2 - U_1}{2^2 - 1} \tag{12}$$

$$V_4 = U_4 + \frac{U_4 - U_2}{2^2 - 1} \tag{13}$$

S and V are more precise approximations with error h^4. Thus the process continues up to calculated R, W with error h^6 and Q with error h^8. Consequent approximated value of the calculated integral is represented by parameter Q:

$$Q_8 = R_8 + \frac{R_8 - R_4}{2^6 - 1} \tag{14}$$

The final value Q is the result of the calculations whose results are written down into the scheme. As the Romberg's method is in reality the numerical calculation of the approximation results for different divided interval steps, it is possible to use another
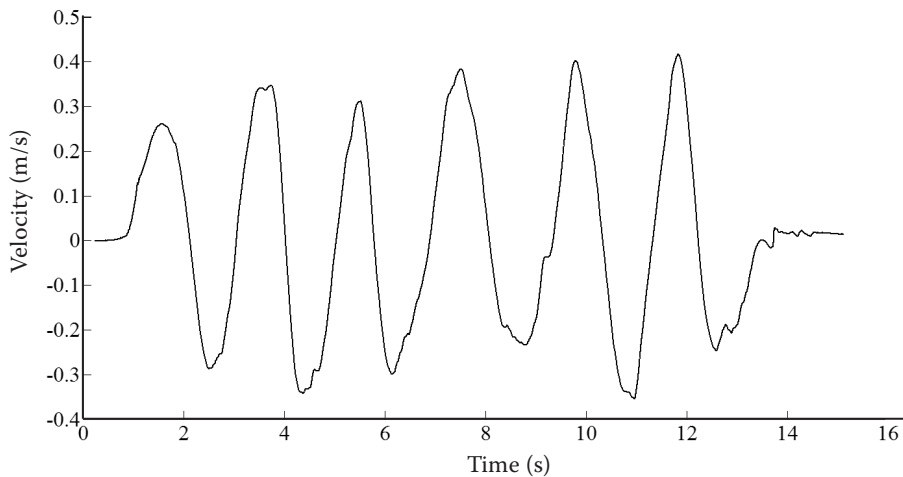
Fig. 3. Velocity course used for our example

approximation instead of the trapezoidal method, for example Simpson's rule. This method is simple but very precise. The errors comparison of the individual integration methods is showed in Fig. 2.

RESULTS AND DISCUSSION

Input acceleration and angular velocity data were measured for fifteen seconds. The velocity curve was optimised to exclude other errors (Fig. 3) and it was controlled by autonomous robot, whose navigation was based on incremental encoders. The calculations were evaluated by MATLAB application (MathWorks Inc., Natick, USA). The sample frequency of 816.6599 Hz and the properly selected course line of velocity for it enabled to minimise the error caused by the content of higher harmonic components in the signal (according to the criterion of Shannon 1949). The acceleration course measured by ADIS16405BLMZ is shown on Fig. 4.

This change can be seen also on the final errors in their decrease. It is due to the selected type of ap-

proximation. It is not possible to describe the continuously modifying common phenomenon and changes by means of the trapezoidal method. The most exact results are obtained if concavity and convexity of the course change and when the positive and negative errors compensate each other.

Slightly better is the use of Simpson's rule in the Romberg's method that uses polynomial approximation, most frequently of the second rank. However, this calculation is more complicated because the final area can not be calculated from two values only, but from several measurements in the selected interval. The deviation course of the calculated velocity to real velocity for the Romberg's method and the trapezoidal approximation refers to the significant difference between the methods listed. The use of Romberg's method with trapezoidal approximation shows a significantly smaller deviation when compared with trapezoidal rule itself.

On Fig. 5, the velocity error course is shown. Romberg's method is caused by the trapezoidal approximation used. In the time interval between the fourteenth and fifteenth seconds the error decreases in Romb-
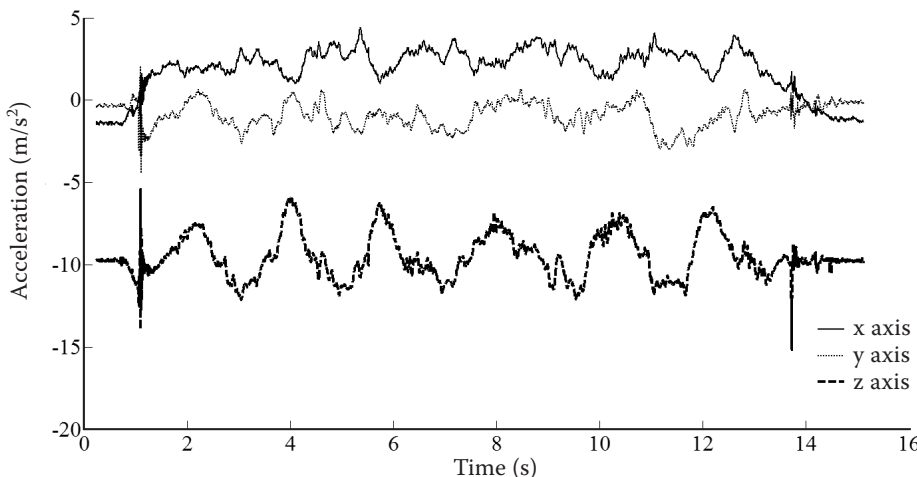

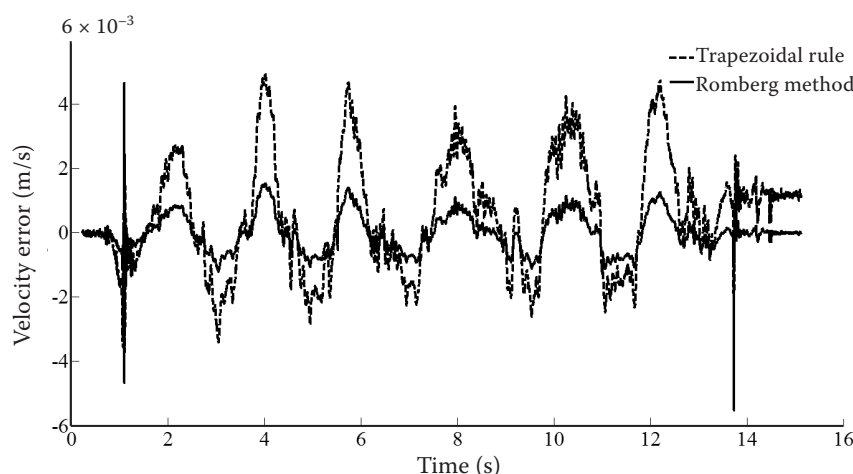
Fig. 4. Acceleration course used for our example

Fig. 5. Velocity error course according to time

erg's method. The trapezoidal rule is inaccurate for the use in the algorithm of strapdown inertial navigation systems. The trapezoidal method velocity error was in this example 1.23×10^{-3} m/s in the fifteenth-second measurement. Romberg's method velocity error was for the same input data 0.16×10^{-3} m/s.

CONCLUSION

The application of numerical integration methods to inertial navigation demands high attention and knowledge of various course specifications of the input values, from which the final integral is calculated. These values are the acceleration and time difference between the individual samples. Acceleration varies continuously, it does not jump. Therefore, it is preferable to use polynomial approximation in connection with an appropriate interpolation method. The reason resides in more precise description of the real functions by parabolas rather than lines. The intent of time belongs to the most exact values measured, therefore the projection of the time error is negligible.

The final actual position is defined by the acceleration calculation onto velocity and the velocity calculation onto trajectory. Acceleration is integrated two times together with the error, which accumulates in time. This error can be eliminated by filtering, but cannot be completely removed. Therefore, it is needed to pay attention to this problem, which should lead to general inertial navigation accuracy improvement. Navigation of higher quality may find very broad utilisation in the future, for example in the storehouses, where danger materials (substances) or heavy machine parts are used.

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Corresponding author:

Ing. Vladimír Cvikovič, Ph.D., Slovak University of Agriculture in Nitra, Faculty of Engineering,
Department of Electrical Engineering, Automation and Informatics, Tr. A. Hlinku 2, 949 76 Nitra, Slovak Republic
phone: + 421 376 415 783, e-mail: vladimir. cvikovic@uniag. sk