Determination of the technical parameters of shaping tools for delimbing head

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Abstract

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It is usually a technical problem to find mathematical ideal shape of knives of delimbing head. Determination of their shape in the form of general curve could lead to the technically unfeasible dimensions of the head. Criterion of the optimal shape of cutting contour of head is tightness of encirclement of cross section of trunk by knives. Research attention was aimed at conics. Parabola is the most suitable curve for achieving the best quality of delimbing.

Keywords: delimber; forest machine; delimbing tool

Solution of an optimal shape of the cutting contour of knives leads to the choice of such kind of equation and to acquiring such numerical values of parameters of this equation for which an approach of tools to the cross-section of stem seams to be the closest.

Basic criteria of evaluation of delimbing quality in practice are the amount of branch reminders and their height. The use of these two criteria leads to forecasting and quality assessment of delimbing (Golod 1987; Barinov, Alexandrov 1988; Voronicyn, Gugelev 1989; Mac Donald 1993; Mikleš, Marko 1993; Mikleš 1994; Mikleš, Mikleš 2005).

The criterion of optimal shape of cutting contour has been an adopted coefficient of trunk tightness encirclement by the cutting contour.

$$K_{op} = \frac{V_1}{V_2} \approx \frac{\sum S_{1j}}{\sum S_{2j}} \leq 1 \qquad (1)$$

where:

K_{op} – coeficient of trunk tightness encirclement by the cutting contour

V_1 – stem volume with the diameter D_{max} at butt and D_{min} at upper cut

V_2 – stem volume circumscribed by cutting contour of knives around the stem

S_{1j} – area of circle with the diameter D_j; $D_{min} \leq D_j \leq D_{max}$

S_{2j} – area circumscribed bordered by cutting contour of knives, which encircles a circle with the diameter D_j

This criterion includes the volume of gap between the stem and cutting contour. As it follows from it, this criterion expresses the number as well as the height of remaining branches after delimbing of the stem. Max. values of the coefficient of tightness (completeness) of encirclement of stem by cutting contour correspond with the best quality of delimbing.

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By inserting of the axis of coordinate system, according to Fig. 1 we will get an area circumscribed by cutting contour described according to equation $y = f_1(x)$, in general form:

$$S_2 = 2n\left[\int_0^{x_1} f_1(x)dx - \frac{x_1 f_1(x_1)}{2}\right] \quad (2)$$

where:

n – number of knives of cutting contour

$x_1, f_1, (x_1)$ – coordinates of intersection points of two adjacent knives

The Eq. (2) holds under the assumption that the other knife does not fall down into the gap between the stem and one of the knives.

In the opposite case it is necessary to use the equation:

$$S_2 = 2n\left[\int_0^{x_1} f_2(x)dx - \frac{x_1 f_1(x_1)}{2}\right] \quad (3)$$

where:

$f_2(x)$ – function whose graph is rotated by 360°C/n to the graph of function $f_1(x)$

To describe the shape of cutting contour the best seems to use the equation of parabola, because by this equation a wide range of axial symmetric curves (circle, ellipse, hyperbola, sinusoid, etc.) can be approximated.

To find mathematical ideal shape of knife – i.e. a general curve k which would create an osculatory (three-point) contact with the stem circle in each position during the stem feed – met with a technical problem. The dimension of the knife should have been technically unfeasible and so the attention has turned to conics. Out of them, the greatest prerequisites to achieve these intentions have the parabola.

MATERIAL AND METHODS

Order of mathematical solution of the task (Rektorys et al. 1981; Jančina, Pekárek 1987; Abramowitz, Stegun 2002; Bronshtein et al. 2004):

a) determination of conditions for the contact of the parabola and the circle – the girth
b) computation of the parabola parameter – the shape of knife so that, in the given number of knives, the "remainder" after delimbing was minimal
c) verification of the condition that the knife has to be "inside" the parabola
d) determination of the number of knives necessary for minimum "reminder" of branches

Determination of the contact of curves – the circle (stem) and the parabola. Mathematical considerations and computations are in a rectangular coordinate system (x, y). Location of the curves according to Fig. 2 (the circles touch the x axis in the tangent line; the centres are on the y axis; the parabola touches the circles).

In this position the equations of the curves are:

Parabola: $$y = -\frac{x^2}{2p} + k \quad (4)$$

where:

p – parameter

k – y coordinate of the vertex of parabola

Circle: $$x^2 + y^2 - 2ry = 0 \quad (5)$$

where:

r – radius of the circle

We are going to solve the system of Eqs (4), (5) with variables x, y and we determine the conditions

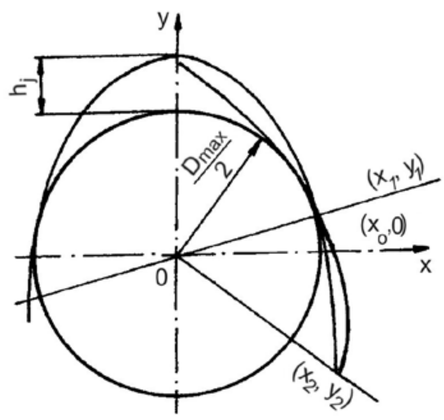

Fig. 1. The shape of the curve of knife feeding trunk

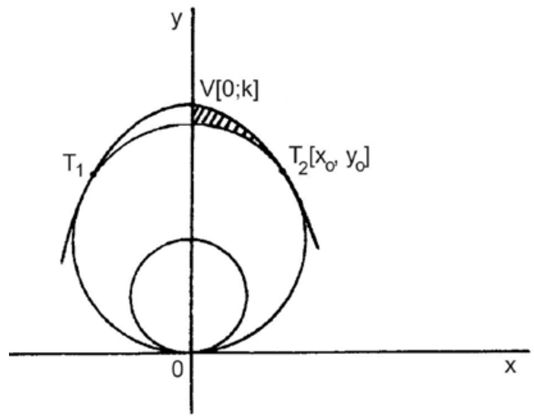

Fig. 2. Location of coordinate system – (x, y)

– relations among r, p, k – for double roots (or even quadruple ones).

$$x^2 + \left(-\frac{x^2}{2p} + k\right)^2 - 2r\left(-\frac{x^2}{2p} + k\right) = 0 \qquad (6)$$

$$x^2 + \frac{x^4}{4p^2} - \frac{kx^2}{p} + k^2 + \frac{rx^2}{p} - 2kr = 0 / 4p^2$$

$$4p^2x^2 + x^4 - 4pkx^2 + 4p^2k^2 + 4prx^2 - 8kp^2r = 0$$

$$x^4 + \left(4p^2 - 4pk + 4pr\right)x^2 + 4p^2k^2 - 8krp^2 = 0 \qquad (7)$$

Thus we have the biquadratic equation – it has 4 roots.

(I) We got double roots, if the discriminant of equation $D = 0$, thus

$$D = 16\left(p^4 + p^2k^2 + p^2r^2 - 2p^3k + 2p^3r - 2p^2kr\right) - 4\left(4p^2k^2 - 8kp^2r\right) =$$
$$= 16\left(p^4 + p^2r^2 - 2p^3k + 2p^3r\right) = 0$$

from which: $k = \frac{(p+r)^2}{2p}$ (8)

and points of contact: $x_{1,2}^2 = \frac{4pk - 4p^2 - 4pr}{2}\ (D = 0)$ (9)

from Eqs (8) and (9) follows $x_1^2 - x_2^2 = r^2 - p^2$ then:

$$x_{11} = x_{21} = \sqrt{r^2 - p^2}$$

$$x_{12} = x_{22} = -\sqrt{r^2 - p^2} \qquad (10)$$

For y coordinates of points of contact Eq. (10) we insert into Eq. (4).

$y_1 = y_2 = r + p$ (11)

So the points of contact are:

$$T_1\left[-\sqrt{r^2 - p^2}; r + p\right]$$

$$T_2\left[\sqrt{r^2 - p^2}; r + p\right] \qquad (12)$$

(see in Fig. 2)

(II) In the case I. the roots of Eq. (10) have significance – they are real – only in the case $r \geq p$.

Therefore we must also search other double roots.

In the case (10) was $x_{11} = x_{21} (x_{12} = x_{22})$.

Let us examine the case: $x_{11} = x_{12}$ (or $i\ x_{21} = x_{22}$).

But x_{11}, x_{12} are the root of equation $x_1^2 = a \Rightarrow$ and so $x_{11} = x_{12} \Rightarrow x_{11} = \sqrt{a}$

$$\sqrt{a} = -\sqrt{a} \qquad x_{12} = -\sqrt{a}$$

$$\sqrt[2]{a} = 0;\ 4a = 0;\ a = 0$$

and so $x_1^2 = 0 \rightarrow$ from which $x_{11} = x_{12} = 0$

Let us also find the conditions for k in this case.

It is obvious from biquadratic Eq. (7)

We got $x_1^2 = 0$ if the constant term of equation is put equal to zero.

Then we get:

$$x^4 + \left(4p^2 - 4pk + 4pr\right)x^2 = 0 \qquad (13)$$

and the condition:

$$4p^2k^2 - 8p^2kr = 0 \qquad (14)$$

from Eq. (14): $k = 2r$ (15)

and really the solution of Eq. (13) is:

$$x^2\left[x^2 + \left(4p^2 - 4pk\right) + 4pr\right] = 0 \qquad (16)$$

(1) $x_1^2 = 0 \rightarrow x_{11} = x_{12} = 0$

Note: We must still prove that the remaining roots are not real – the parabola apart from the contact should also intersect the circle in two points:

So from Eq. (16) further:

(2) $x^2 + 4p^2 - 4pk + 4pr = 0$ (17)

after insertion from Eq. (15) to Eq. (17) we get:

$$x_{22}^2 = 4p(r - p) \qquad (18)$$

if x_{21}, x_{22} were imaginary, it must obviously hold in Eq. (18): ($r > 0$, $p > 0$)

$$r - p < 0 \Rightarrow r < p \qquad (19)$$

and so the case: $r < p$ leads to the contact – the result: the point of contact T [0, 2] (see Fig. 3).

Summary:

in case if $p \leq r$ (the parameter p does not change, radius of a circle – the stem changes) for the contact is:

$$k = \frac{(p+r)^2}{2p}$$

Points of contact points T_i have coordinates

$$T_i\left[\pm\sqrt{r^2 p^2}; r + p\right]\ i = 1, 2$$

and the equation of the parabola is

$$y = \frac{x^2}{2p} + \frac{(p+r)^2}{2p} \qquad (20)$$

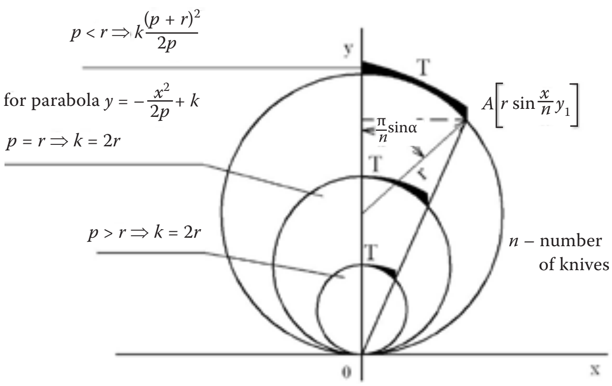

Fig. 3. Changing touch of the point T knife and trunk

in case, if $p > r$, for the contact has to be: $k = 2r$

The point of contact is T [0; 2r] and the equation of the parabola is:

$$y = \frac{x^2}{2p} + 2r \quad (21)$$

Determination of the size of parabola parameter. Now we need to express the magnitude of the area of "remainder" after delimbing on one place of stem (it is a place demarcated by the parabola and the circle – for a chosen number of knives: n we take the area of the half of one knife).

Let us consider the number of knives n (n = 2, 3, ...).

Domains of integration (computation of area) – considering the half of knife obviously are:

$$\langle 0; r\sin\frac{\pi}{n}\rangle \quad (22)$$

Note: In the next computation we will use the range of stem diameters from 10 to 64 cm (thus: $r_1 = 5$ up to $r_2 = 32$).

The marked area in the case I is:

$$\int_0^{r\sin\frac{\pi}{n}} \left[-\frac{x^2}{2p} + \frac{(p+r)^2}{2p} \left(r + \sqrt{r^2 - x^2} \right) \right] dx$$

$$\left(y = -\frac{x^2}{2p} + \frac{(p+r)^2}{2p};\ y = r + \sqrt{r^2 - x^2} \right)$$

thus $$P_1 = \int_0^{r\sin\frac{\pi}{n}} \left(\frac{p^2 + r^2 - x^2}{2p} - \sqrt{r^2 - x^2} \right) dx \quad (22a)$$

for the case II:

parabola: $y = \frac{x^2}{2p} + 2r$; circle $y = r + \sqrt{r^2 - x^2}$

$$P_2 = \int_0^{r\sin\frac{\pi}{n}} \left(r - \frac{x^2}{2p} - \sqrt{r^2 - x^2} \right) dx \quad (22b)$$

"Sum" of all areas in the case I:

$$S_1 = \int_P^{32} \int_0^{r\sin\frac{\pi}{n}} \left(\frac{p^2 + r^2 - x^2}{2p} - \sqrt{r^2 - x^2} \right) dx dr \quad (23a)$$

"Sum" of all areas in the case II:

$$S_2 = \int_5^{P} \int_0^{r\sin\frac{\pi}{n}} \left(r - \frac{x^2}{2p} - \sqrt{r^2 - x^2} \right) dx dr \quad (23b)$$

Total "sum" is $S = S_1 + S_2$

After computation of these double integrals in Eq. (23a) and Eq. (23b) we get the function of the parabola parameter p and number of knives n: $S = f(p, n)$,

$$f(p,n) = -\frac{1}{24}\sin\frac{\pi}{n} p^3 + 256 \times \sin\frac{\pi}{n} p + \left(131{,}072 \times \sin\frac{\pi}{n} - \frac{349{,}317}{8} \times \sin^3\frac{\pi}{n} \right) \times \frac{1}{p} \frac{10{,}881}{2} \times \frac{\pi}{n} - \frac{10{,}881}{4} \sin\frac{2\pi}{n} - \frac{125}{3} \times \sin\frac{\pi}{n} \quad (24)$$

By differentiate of Eq. (24) with respect to variable p (n is a constant) and by searching for the local minimum of function we have got:

The function has its minimal point Eq. (24) for chosen n (n = 2, 3, 4, ...) for

$$p = \sqrt{\frac{2{,}048 - 1{,}397{,}268 \times \sin\frac{\pi}{n}}{2}} \quad (25)$$

Considering some technically feasible number of knives n, the optimum values of knife parameters – the parabola are: $n = 2 \rightarrow p = 20.808$; $n = 3 \rightarrow p = 22.632$; $n = 4 \rightarrow p = 24.62$; $n = 5 \rightarrow p = 26.01$; $n = 6 \rightarrow p = 26.99$

Determination of condition for engagement point of knife. The contact of knife must be "inside" the parabola – the "engagement" of knife.

That is – x coordinant of the point of contact T_2 – must be smaller than the upper limit of interval for certain integral – "area" of the half of knife.

Therefore, according to Eq. (12):

$$\sqrt{r^2 - p^2} < r\sin\frac{\pi}{n} \quad (26)$$

for: $r_{max} = 32$, $r_{min} = p_{opt}$ ($p_{opt} < 32$) ($r > 0$, $p > 0$)

From Eq. (26) we get:

$r^2 - p^2 < r^2 \sin\frac{\pi}{n}$ and so $r^2 \cos^2\frac{\pi}{n} < p_{opt}^2$

That is for $r > 0, \cos\frac{\pi}{n} > 0, p > 0$ is $r\cos\frac{\pi}{n} < p_{opt}$

This expression is valid for n = 2, 3, 4, 5, which can easily be proved, if we take into account the expression of Eq. (25).

Determination of minimum area (remainder) after delimbing. The overall „remainder" after delimbing the whole stem will be obvious – grand total sum S multiplied by 2 × n.

That is $S_u = S \times 2n$ (27)

where:

S_u – "remainder" (volume) after delimbing

Now it still arises a question, at which n – number of knives, will be the function $S = f(p, n)$, and consequently also the function $S_u = 2 \times n \times S$, with optimal p minimal?

We have found by computation that the function $f(p, n)$, if the variable is n, the parameter p is constant – is decreasing – with an increase in number of knives n the S, and also S_u – are decreasing – that is the "remainder" after delimbing is:

At $n = 2$ and optimal p_{opt} is $S_u \doteq 2{,}258\ cm^3$
$n = 3$ p_{opt} is $S_u \doteq 1{,}634\ cm^3$
$n = 4$ p_{opt} is $S_u \doteq 1{,}053\ cm^3$
$n = 5$ p_{opt} is $S_u \doteq 739\ cm^3$

To decide about the number of knives is then only a technical problem.

Note: With increasing $n > 5$ it is necessary to increase also the parameter p.

CONCLUSION

From the theoretic analyses follows that the system of knives advancing around the stem with the same number of knives works in more satisfactory way than the system of tilting (swivelling). The greatest shortcoming is difficult insertion of the stem end into the head opening. With the models of mobile machines the used delimbing heads are mostly with tilting knives. The heads of harvesters and processors use to have odd number of knives and central knife is usually fixed. The tightness of encirclement of knives to the stem in the full range of delimbed diameters is acquired by bending of knives in the curve shape, which is most often conic (parabola). The goal of the paper was to decide on the method of determining the parameters of shape (curve) of the knife and their number along with ensuring max. quality of delimbing.

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